

## Index sets

$$HH = \{1, 2\}$$

$$SEC = \{A, B, C\}$$

## 1 HOUSEHOLD $h \in HH$

### 1.1 Optimisation problem

$$\max_{(D^{(h,s)})_{s \in SEC}} U^{(h)} = \left( \sum_{s \in SEC} \alpha^{(h,s)} D^{(h,s)} \omega^{-1(-1+\omega)} \right)^{\omega(-1+\omega)^{-1}} \quad (1.1)$$

s.t. :

$$\sum_{s \in SEC} p^{(s)} D^{(h,s)} = L^{(h)} + \phi^{(h)} \left( \sum_{s \in SEC} \pi^{(s)} \right) + p^k K^{(h)} \left( \lambda^{\text{HOUSEHOLD}^1(h)} \right) \quad (1.2)$$

### 1.2 Identities

$$hi \in HH: \quad K^{(hi)} = pr^k(hi) \quad (1.3)$$

$$hi \in HH: \quad L^{(hi)} = pr^1(hi) \quad (1.4)$$

### 1.3 First order conditions

$$s \in SEC: \quad -\lambda^{\text{HOUSEHOLD}^1(h)} p^{(s)} + \alpha^{(h,s)} D^{(h,s)^{-1+\omega^{-1}(-1+\omega)}} \left( \sum_{s \in SEC} \alpha^{(h,s)} D^{(h,s)} \omega^{-1(-1+\omega)} \right)^{-1+\omega(-1+\omega)^{-1}} = 0 \quad (D^{(h,s)}) \quad (1.5)$$

## 2 FIRM $s \in SEC$

### 2.1 Optimisation problem

$$\max_{Y^{(s)}, K^{(s)}, L^{(s)}, (X^{(s, \dot{s}i}))_{\dot{s}i \in SEC}} \pi^{(s)} = -L^{(s)} - p^k K^{(s)} + p^{(s)} Y^{(s)} - \sum_{\dot{s}i \in SEC} p^{(\dot{s}i)} X^{(s, \dot{s}i)} \quad (2.1)$$

s.t. :

$$Y^{(s)} = \gamma^{(s)} K^{(s)\beta^k(s)} L^{(s)\beta^l(s)} \left( \prod_{\dot{s}i \in SEC} X^{(s, \dot{s}i)\beta^x(s, \dot{s}i)} \right) \left( \lambda^{\text{FIRM}^1(s)} \right) \quad (2.2)$$

### 2.2 First order conditions

$$-\lambda^{\text{FIRM}^1(s)} + p^{(s)} = 0 \quad (Y^{(s)}) \quad (2.3)$$

$$-p^k + \beta^k(s) \gamma^{(s)} \lambda^{\text{FIRM}^1(s)} K^{(s)-1+\beta^k(s)} L^{(s)\beta^l(s)} \left( \prod_{\dot{s}i \in SEC} X^{(s, \dot{s}i)\beta^x(s, \dot{s}i)} \right) = 0 \quad (K^{(s)}) \quad (2.4)$$

$$-1 + \beta^l(s) \gamma^{(s)} \lambda^{\text{FIRM}^1(s)} K^{(s)\beta^k(s)} L^{(s)-1+\beta^l(s)} \left( \prod_{\dot{s}i \in SEC} X^{(s, \dot{s}i)\beta^x(s, \dot{s}i)} \right) = 0 \quad (L^{(s)}) \quad (2.5)$$

$$\dot{s}i \in SEC: \quad -p^{(\dot{s}i)} + \beta^x(s, \dot{s}i) \gamma^{(s)} \lambda^{\text{FIRM}^1(s)} X^{(s, \dot{s}i)-1} K^{(s)\beta^k(s)} L^{(s)\beta^l(s)} \left( \prod_{\dot{s}i' \in SEC} X^{(s, \dot{s}i')\beta^x(s, \dot{s}i')} \right) = 0 \quad (X^{(s, \dot{s}i)}) \quad (2.6)$$

### 2.3 First order conditions after reduction

$$-p^k + \beta^k(s) \gamma^{(s)} p^{(s)} K^{(s)-1+\beta^k(s)} L^{(s)\beta^l(s)} \left( \prod_{\dot{s}i \in SEC} X^{(s, \dot{s}i)\beta^x(s, \dot{s}i)} \right) = 0 \quad (K^{(s)}) \quad (2.7)$$

$$-1 + \beta^l(s) \gamma^{(s)} p^{(s)} K^{(s)\beta^k(s)} L^{(s)-1+\beta^l(s)} \left( \prod_{\dot{s}i \in SEC} X^{(s, \dot{s}i)\beta^x(s, \dot{s}i)} \right) = 0 \quad (L^{(s)}) \quad (2.8)$$

$$\dot{s}i \in SEC: \quad -p^{(\dot{s}i)} + \beta^x(s, \dot{s}i) \gamma^{(s)} p^{(s)} X^{(s, \dot{s}i)-1} K^{(s)\beta^k(s)} L^{(s)\beta^l(s)} \left( \prod_{\dot{s}i' \in SEC} X^{(s, \dot{s}i')\beta^x(s, \dot{s}i')} \right) = 0 \quad \left( (X^{(s, \dot{s}i)})_{\dot{s}i \in SEC} \right) \quad (2.9)$$

### 3 EQUILIBRIUM

#### 3.1 Identities

$$s \in SEC: \quad Y^{(s)} = \sum_{h \in HH} D^{(h,s)} + \sum_{\tilde{s} \in SEC} X^{(s,\tilde{s})} \quad (3.1)$$

$$\sum_{h \in HH} L^{(h)} = \sum_{s \in SEC} L^{(s)} \quad (3.2)$$

#### 4 Equilibrium relationships (before expansion and reduction)

$$- \sum_{h \in HH} L^{(h)} + \sum_{s \in SEC} L^{(s)} = 0 \quad (4.1)$$

$$h \in HH: \quad \mu^{k(h)} - K^{(h)} = 0 \quad (4.2)$$

$$h \in HH: \quad \mu^{1(h)} - L^{(h)} = 0 \quad (4.3)$$

$$h \in HH: \quad U^{(h)} - \left( \sum_{s \in SEC} \alpha^{(h,s)} D^{(h,s)} \omega^{-1(-1+\omega)} \right)^{\omega(-1+\omega)^{-1}} = 0 \quad (4.4)$$

$$h \in HH: \quad L^{(h)} + \phi^{(h)} \left( \sum_{s \in SEC} \pi^{(s)} \right) + p^k K^{(h)} - \sum_{s \in SEC} p^{(s)} D^{(h,s)} = 0 \quad (4.5)$$

$$h \in HH: \quad s \in SEC: \quad -\lambda^{\text{HOUSEHOLD}^1(h)} p^{(s)} + \alpha^{(h,s)} D^{(h,s)^{-1+\omega^{-1}(-1+\omega)}} \left( \sum_{s \in SEC} \alpha^{(h,s)} D^{(h,s)} \omega^{-1(-1+\omega)} \right)^{-1+\omega(-1+\omega)^{-1}} = 0 \quad (4.6)$$

$$s \in SEC: \quad -1 + \beta^{1(s)} \gamma^{(s)} p^{(s)} K^{(s)\beta^{k(s)}} L^{(s)^{-1+\beta^{1(s)}}} \left( \prod_{\tilde{s} \in SEC} X^{(s,\tilde{s})\beta^{x(s,\tilde{s})}} \right) = 0 \quad (4.7)$$

$$s \in SEC: \quad -p^k + \beta^{k(s)} \gamma^{(s)} p^{(s)} K^{(s)^{-1+\beta^{k(s)}}} L^{(s)\beta^{1(s)}} \left( \prod_{\tilde{s} \in SEC} X^{(s,\tilde{s})\beta^{x(s,\tilde{s})}} \right) = 0 \quad (4.8)$$

$$s \in SEC: \quad -Y^{(s)} + \gamma^{(s)} K^{(s)\beta^{k(s)}} L^{(s)\beta^{1(s)}} \left( \prod_{\tilde{s} \in SEC} X^{(s,\tilde{s})\beta^{x(s,\tilde{s})}} \right) = 0 \quad (4.9)$$

$$s \in SEC: \quad -Y^{(s)} + \sum_{h \in HH} D^{(h,s)} + \sum_{\tilde{s} \in SEC} X^{(s,\tilde{s})} = 0 \quad (4.10)$$

$$s \in SEC: \quad \pi^{(s)} + L^{(s)} + p^k K^{(s)} - p^{(s)} Y^{(s)} + \sum_{\mathbf{si} \in SEC} p^{(\mathbf{si})} X^{(s, \mathbf{si})} = 0 \quad (4.11)$$

$$s \in SEC: \quad \mathbf{si} \in SEC: \quad -p^{(\mathbf{si})} + \beta^{x(s, \mathbf{si})} \gamma^{(s)} p^{(s)} X^{(s, \mathbf{si})^{-1}} K^{(s)} \beta^{k(s)} L^{(s)} \beta^{l(s)} \left( \prod_{\mathbf{si}' \in SEC} X^{(s, \mathbf{si}')} \beta^{x(s, \mathbf{si}')} \right) = 0 \quad (4.12)$$

## 5 Equilibrium relationships (after expansion and reduction)

$$-1 + \beta^{l(A)} \gamma^{(A)} p^{(A)} K^{(A)} \beta^{k(A)} L^{(A)}^{-1 + \beta^{l(A)}} X^{(A,A)} \beta^{x(A,A)} X^{(A,B)} \beta^{x(A,B)} X^{(A,C)} \beta^{x(A,C)} = 0 \quad (5.1)$$

$$-1 + \beta^{l(B)} \gamma^{(B)} p^{(B)} K^{(B)} \beta^{k(B)} L^{(B)}^{-1 + \beta^{l(B)}} X^{(B,A)} \beta^{x(B,A)} X^{(B,B)} \beta^{x(B,B)} X^{(B,C)} \beta^{x(B,C)} = 0 \quad (5.2)$$

$$-1 + \beta^{l(C)} \gamma^{(C)} p^{(C)} K^{(C)} \beta^{k(C)} L^{(C)}^{-1 + \beta^{l(C)}} X^{(C,A)} \beta^{x(C,A)} X^{(C,B)} \beta^{x(C,B)} X^{(C,C)} \beta^{x(C,C)} = 0 \quad (5.3)$$

$$p^{k(1)} - K^{(1)} = 0 \quad (5.4)$$

$$p^{k(2)} - K^{(2)} = 0 \quad (5.5)$$

$$p^{l(1)} - L^{(1)} = 0 \quad (5.6)$$

$$p^{l(2)} - L^{(2)} = 0 \quad (5.7)$$

$$-p^k + \beta^{k(A)} \gamma^{(A)} p^{(A)} K^{(A)}^{-1 + \beta^{k(A)}} L^{(A)} \beta^{l(A)} X^{(A,A)} \beta^{x(A,A)} X^{(A,B)} \beta^{x(A,B)} X^{(A,C)} \beta^{x(A,C)} = 0 \quad (5.8)$$

$$-p^k + \beta^{k(B)} \gamma^{(B)} p^{(B)} K^{(B)}^{-1 + \beta^{k(B)}} L^{(B)} \beta^{l(B)} X^{(B,A)} \beta^{x(B,A)} X^{(B,B)} \beta^{x(B,B)} X^{(B,C)} \beta^{x(B,C)} = 0 \quad (5.9)$$

$$-p^k + \beta^{k(C)} \gamma^{(C)} p^{(C)} K^{(C)}^{-1 + \beta^{k(C)}} L^{(C)} \beta^{l(C)} X^{(C,A)} \beta^{x(C,A)} X^{(C,B)} \beta^{x(C,B)} X^{(C,C)} \beta^{x(C,C)} = 0 \quad (5.10)$$

$$-p^{(A)} + \beta^{x(A,A)} \gamma^{(A)} p^{(A)} X^{(A,A)}^{-1} K^{(A)} \beta^{k(A)} L^{(A)} \beta^{l(A)} X^{(A,A)} \beta^{x(A,A)} X^{(A,B)} \beta^{x(A,B)} X^{(A,C)} \beta^{x(A,C)} = 0 \quad (5.11)$$

$$-p^{(A)} + \beta^{x(B,A)} \gamma^{(B)} p^{(B)} X^{(B,A)}^{-1} K^{(B)} \beta^{k(B)} L^{(B)} \beta^{l(B)} X^{(B,A)} \beta^{x(B,A)} X^{(B,B)} \beta^{x(B,B)} X^{(B,C)} \beta^{x(B,C)} = 0 \quad (5.12)$$

$$-p^{(A)} + \beta^{x(C,A)} \gamma^{(C)} p^{(C)} X^{(C,A)}^{-1} K^{(C)} \beta^{k(C)} L^{(C)} \beta^{l(C)} X^{(C,A)} \beta^{x(C,A)} X^{(C,B)} \beta^{x(C,B)} X^{(C,C)} \beta^{x(C,C)} = 0 \quad (5.13)$$

$$-p^{(B)} + \beta^{x(A,B)} \gamma^{(A)} p^{(A)} X^{(A,B)}^{-1} K^{(A)} \beta^{k(A)} L^{(A)} \beta^{l(A)} X^{(A,A)} \beta^{x(A,A)} X^{(A,B)} \beta^{x(A,B)} X^{(A,C)} \beta^{x(A,C)} = 0 \quad (5.14)$$

$$-p^{(B)} + \beta^{x(B,B)} \gamma^{(B)} p^{(B)} X^{(B,B)}^{-1} K^{(B)} \beta^{k(B)} L^{(B)} \beta^{l(B)} X^{(B,A)} \beta^{x(B,A)} X^{(B,B)} \beta^{x(B,B)} X^{(B,C)} \beta^{x(B,C)} = 0 \quad (5.15)$$

$$-p^{(B)} + \beta^{x(C,B)} \gamma^{(C)} p^{(C)} X^{(C,B)}^{-1} K^{(C)} \beta^{k(C)} L^{(C)} \beta^{l(C)} X^{(C,A)} \beta^{x(C,A)} X^{(C,B)} \beta^{x(C,B)} X^{(C,C)} \beta^{x(C,C)} = 0 \quad (5.16)$$

$$-p^{(C)} + \beta^{x(A,C)} \gamma^{(A)} p^{(A)} X^{(A,C)-1} K^{(A)\beta^{k(A)}} L^{(A)\beta^{1(A)}} X^{(A,A)\beta^{x(A,A)}} X^{(A,B)\beta^{x(A,B)}} X^{(A,C)\beta^{x(A,C)}} = 0 \quad (5.17)$$

$$-p^{(C)} + \beta^{x(B,C)} \gamma^{(B)} p^{(B)} X^{(B,C)-1} K^{(B)\beta^{k(B)}} L^{(B)\beta^{1(B)}} X^{(B,A)\beta^{x(B,A)}} X^{(B,B)\beta^{x(B,B)}} X^{(B,C)\beta^{x(B,C)}} = 0 \quad (5.18)$$

$$-p^{(C)} + \beta^{x(C,C)} \gamma^{(C)} p^{(C)} X^{(C,C)-1} K^{(C)\beta^{k(C)}} L^{(C)\beta^{1(C)}} X^{(C,A)\beta^{x(C,A)}} X^{(C,B)\beta^{x(C,B)}} X^{(C,C)\beta^{x(C,C)}} = 0 \quad (5.19)$$

$$U^{(1)} - \left( \alpha^{(1,A)} D^{(1,A)\omega^{-1}(-1+\omega)} + \alpha^{(1,B)} D^{(1,B)\omega^{-1}(-1+\omega)} + \alpha^{(1,C)} D^{(1,C)\omega^{-1}(-1+\omega)} \right)^{\omega(-1+\omega)^{-1}} = 0 \quad (5.20)$$

$$U^{(2)} - \left( \alpha^{(2,A)} D^{(2,A)\omega^{-1}(-1+\omega)} + \alpha^{(2,B)} D^{(2,B)\omega^{-1}(-1+\omega)} + \alpha^{(2,C)} D^{(2,C)\omega^{-1}(-1+\omega)} \right)^{\omega(-1+\omega)^{-1}} = 0 \quad (5.21)$$

$$-Y^{(A)} + \gamma^{(A)} K^{(A)\beta^{k(A)}} L^{(A)\beta^{1(A)}} X^{(A,A)\beta^{x(A,A)}} X^{(A,B)\beta^{x(A,B)}} X^{(A,C)\beta^{x(A,C)}} = 0 \quad (5.22)$$

$$-Y^{(B)} + \gamma^{(B)} K^{(B)\beta^{k(B)}} L^{(B)\beta^{1(B)}} X^{(B,A)\beta^{x(B,A)}} X^{(B,B)\beta^{x(B,B)}} X^{(B,C)\beta^{x(B,C)}} = 0 \quad (5.23)$$

$$-Y^{(C)} + \gamma^{(C)} K^{(C)\beta^{k(C)}} L^{(C)\beta^{1(C)}} X^{(C,A)\beta^{x(C,A)}} X^{(C,B)\beta^{x(C,B)}} X^{(C,C)\beta^{x(C,C)}} = 0 \quad (5.24)$$

$$-\lambda^{\text{HOUSEHOLD}^1(1)} p^{(A)} + \alpha^{(1,A)} D^{(1,A)-1+\omega^{-1}(-1+\omega)} \left( \alpha^{(1,A)} D^{(1,A)\omega^{-1}(-1+\omega)} + \alpha^{(1,B)} D^{(1,B)\omega^{-1}(-1+\omega)} + \alpha^{(1,C)} D^{(1,C)\omega^{-1}(-1+\omega)} \right)^{-1+\omega(-1+\omega)^{-1}} = 0 \quad (5.25)$$

$$-\lambda^{\text{HOUSEHOLD}^1(1)} p^{(B)} + \alpha^{(1,B)} D^{(1,B)-1+\omega^{-1}(-1+\omega)} \left( \alpha^{(1,A)} D^{(1,A)\omega^{-1}(-1+\omega)} + \alpha^{(1,B)} D^{(1,B)\omega^{-1}(-1+\omega)} + \alpha^{(1,C)} D^{(1,C)\omega^{-1}(-1+\omega)} \right)^{-1+\omega(-1+\omega)^{-1}} = 0 \quad (5.26)$$

$$-\lambda^{\text{HOUSEHOLD}^1(1)} p^{(C)} + \alpha^{(1,C)} D^{(1,C)-1+\omega^{-1}(-1+\omega)} \left( \alpha^{(1,A)} D^{(1,A)\omega^{-1}(-1+\omega)} + \alpha^{(1,B)} D^{(1,B)\omega^{-1}(-1+\omega)} + \alpha^{(1,C)} D^{(1,C)\omega^{-1}(-1+\omega)} \right)^{-1+\omega(-1+\omega)^{-1}} = 0 \quad (5.27)$$

$$-\lambda^{\text{HOUSEHOLD}^1(2)} p^{(A)} + \alpha^{(2,A)} D^{(2,A)-1+\omega^{-1}(-1+\omega)} \left( \alpha^{(2,A)} D^{(2,A)\omega^{-1}(-1+\omega)} + \alpha^{(2,B)} D^{(2,B)\omega^{-1}(-1+\omega)} + \alpha^{(2,C)} D^{(2,C)\omega^{-1}(-1+\omega)} \right)^{-1+\omega(-1+\omega)^{-1}} = 0 \quad (5.28)$$

$$-\lambda^{\text{HOUSEHOLD}^1(2)} p^{(B)} + \alpha^{(2,B)} D^{(2,B)-1+\omega^{-1}(-1+\omega)} \left( \alpha^{(2,A)} D^{(2,A)\omega^{-1}(-1+\omega)} + \alpha^{(2,B)} D^{(2,B)\omega^{-1}(-1+\omega)} + \alpha^{(2,C)} D^{(2,C)\omega^{-1}(-1+\omega)} \right)^{-1+\omega(-1+\omega)^{-1}} = 0 \quad (5.29)$$

$$-\lambda^{\text{HOUSEHOLD}^1(2)} p^{(C)} + \alpha^{(2,C)} D^{(2,C)-1+\omega^{-1}(-1+\omega)} \left( \alpha^{(2,A)} D^{(2,A)\omega^{-1}(-1+\omega)} + \alpha^{(2,B)} D^{(2,B)\omega^{-1}(-1+\omega)} + \alpha^{(2,C)} D^{(2,C)\omega^{-1}(-1+\omega)} \right)^{-1+\omega(-1+\omega)^{-1}} = 0 \quad (5.30)$$

$$-L^{(1)} - L^{(2)} + L^{(A)} + L^{(B)} + L^{(C)} = 0 \quad (5.31)$$

$$D^{(1,A)} + D^{(2,A)} + X^{(A,A)} + X^{(B,A)} + X^{(C,A)} - Y^{(A)} = 0 \quad (5.32)$$

$$D^{(1,B)} + D^{(2,B)} + X^{(A,B)} + X^{(B,B)} + X^{(C,B)} - Y^{(B)} = 0 \quad (5.33)$$

$$D^{(1,C)} + D^{(2,C)} + X^{(A,C)} + X^{(B,C)} + X^{(C,C)} - Y^{(C)} = 0 \quad (5.34)$$

$$L^{(1)} + \phi^{(1)} \left( \pi^{(A)} + \pi^{(B)} + \pi^{(C)} \right) + p^k K^{(1)} - p^{(A)} D^{(1,A)} - p^{(B)} D^{(1,B)} - p^{(C)} D^{(1,C)} = 0 \quad (5.35)$$

$$L^{(2)} + \phi^{(2)} \left( \pi^{(A)} + \pi^{(B)} + \pi^{(C)} \right) + p^k K^{(2)} - p^{(A)} D^{(2,A)} - p^{(B)} D^{(2,B)} - p^{(C)} D^{(2,C)} = 0 \quad (5.36)$$

$$\pi^{(A)} + L^{(A)} + p^k K^{(A)} + p^{(A)} X^{(A,A)} - p^{(A)} Y^{(A)} + p^{(B)} X^{(A,B)} + p^{(C)} X^{(A,C)} = 0 \quad (5.37)$$

$$\pi^{(B)} + L^{(B)} + p^k K^{(B)} + p^{(A)} X^{(B,A)} + p^{(B)} X^{(B,B)} - p^{(B)} Y^{(B)} + p^{(C)} X^{(B,C)} = 0 \quad (5.38)$$

$$\pi^{(C)} + L^{(C)} + p^k K^{(C)} + p^{(A)} X^{(C,A)} + p^{(B)} X^{(C,B)} + p^{(C)} X^{(C,C)} - p^{(C)} Y^{(C)} = 0 \quad (5.39)$$

## 6 Equilibrium values

	Equilibrium value
$p^k$	1.0008
$\lambda^{\text{HOUSEHOLD}^{1(1)}}$	0.2524
$\lambda^{\text{HOUSEHOLD}^{1(2)}}$	0.2524
$p^{(A)}$	0.992
$p^{(B)}$	0.9931
$p^{(C)}$	0.9908
$\pi^{(A)}$	-0.0699
$\pi^{(B)}$	-0.06
$\pi^{(C)}$	-0.07
$D^{(1,A)}$	11.2953
$D^{(1,B)}$	3.7712
$D^{(1,C)}$	15.155
$D^{(2,A)}$	18.7964
$D^{(2,B)}$	6.2757
$D^{(2,C)}$	25.2192
$K^{(1)}$	20
$K^{(2)}$	20
$K^{(A)}$	19.9764
$K^{(B)}$	10.0161
$K^{(C)}$	10.0075
$L^{(1)}$	10
$L^{(2)}$	30
$L^{(A)}$	9.9962
$L^{(B)}$	19.9883
$L^{(C)}$	10.0155
$U^{(1)}$	7.5639
$U^{(2)}$	12.5869
$X^{(A,A)}$	10.0764
$X^{(A,B)}$	20.1315
$X^{(A,C)}$	10.0891
$X^{(B,A)}$	10.1046
$X^{(B,B)}$	10.0939
$X^{(B,C)}$	10.1173
$X^{(C,A)}$	20.1917
$X^{(C,B)}$	20.1703
$X^{(C,C)}$	10.1086
$Y^{(A)}$	70.4644
$Y^{(B)}$	60.4427
$Y^{(C)}$	70.6892