

1 CONSUMER

1.1 Optimisation problem

$$\max_{K_t^s, C_t, I_t} U_t = \beta \left(\mathbb{E}_t \left[U_{t+1}^{1-\theta^{EZ}} \right] \right)^{(1-\theta^{EZ})^{-1}} + (-1 + C_t^{1-\eta}) (1-\eta)^{-1} \quad (1.1)$$

s.t. :

$$C_t + I_t = \pi_t + K_{t-1}^s r_t + L_t^s W_t \quad \left(\lambda_t^{\text{CONSUMER}^1} \right) \quad (1.2)$$

$$K_t^s = I_t + K_{t-1}^s (1-\delta) \quad \left(\lambda_t^{\text{CONSUMER}^2} \right) \quad (1.3)$$

1.2 Identities

$$L_t^s = 1 \quad (1.4)$$

1.3 First order conditions

$$-\lambda_t^{\text{CONSUMER}^U} + \beta q_{t-1}^{\text{CONSUMER}^1} (1-\theta^{EZ})^{-1} U_t^{-\theta^{EZ}} = 0 \quad (U_t) \quad (1.5)$$

$$-\lambda_t^{\text{CONSUMER}^2} + \mathbb{E}_t \left[\lambda_{t+1}^{\text{CONSUMER}^U} \left(\lambda_{t+1}^{\text{CONSUMER}^1} r_{t+1} + \lambda_{t+1}^{\text{CONSUMER}^2} (1-\delta) \right) \right] = 0 \quad (K_t^s) \quad (1.6)$$

$$-\lambda_t^{\text{CONSUMER}^1} + C_t^{-\eta} = 0 \quad (C_t) \quad (1.7)$$

$$-\lambda_t^{\text{CONSUMER}^1} + \lambda_t^{\text{CONSUMER}^2} = 0 \quad (I_t) \quad (1.8)$$

2 FIRM

2.1 Optimisation problem

$$\max_{K_t^d, L_t^d, Y_t} \pi_t = Y_t - L_t^d W_t - r_t K_t^d \quad (2.1)$$

s.t. :

$$Y_t = Z_t K_t^{d\alpha} L_t^{d^{1-\alpha}} \quad \left(\lambda_t^{\text{FIRM}^1} \right) \quad (2.2)$$

2.2 First order conditions

$$-r_t + \alpha \lambda_t^{\text{FIRM}^1} Z_t K_t^{d-1+\alpha} L_t^{d^{1-\alpha}} = 0 \quad (K_t^d) \quad (2.3)$$

$$-W_t + \lambda_t^{\text{FIRM}^1} Z_t (1-\alpha) K_t^{d\alpha} L_t^{d-\alpha} = 0 \quad (L_t^d) \quad (2.4)$$

$$1 - \lambda_t^{\text{FIRM}^1} = 0 \quad (Y_t) \quad (2.5)$$

2.3 First order conditions after reduction

$$-r_t + \alpha Z_t K_t^{d-1+\alpha} L_t^{d^{1-\alpha}} = 0 \quad (K_t^d) \quad (2.6)$$

$$-W_t + Z_t (1-\alpha) K_t^{d\alpha} L_t^{d-\alpha} = 0 \quad (L_t^d) \quad (2.7)$$

3 EQUILIBRIUM

3.1 Identities

$$K_t^d = K_{t-1}^s \quad (3.1)$$

$$L_t^d = L_t^s \quad (3.2)$$

4 EXOG

4.1 Identities

$$Z_t = e^{\epsilon_t^Z + \phi \log Z_{t-1}} \quad (4.1)$$

5 Equilibrium relationships (after reduction)

$$q_t^{\text{CONSUMER}^1} - E_t \left[U_{t+1}^{1-\theta^{\text{EZ}}} \right] = 0 \quad (5.1)$$

$$-r_t + \alpha Z_t 1^{1-\alpha} K_{t-1}^s{}^{-1+\alpha} = 0 \quad (5.2)$$

$$-W_t + Z_t (1-\alpha) 1^{-\alpha} K_{t-1}^s{}^\alpha = 0 \quad (5.3)$$

$$-Y_t + Z_t 1^{1-\alpha} K_{t-1}^s{}^\alpha = 0 \quad (5.4)$$

$$-Z_t + e^{\epsilon_t^Z + \phi \log Z_{t-1}} = 0 \quad (5.5)$$

$$\beta q_t^{\text{CONSUMER}^1}{}^{-1+(1-\theta^{\text{EZ}})^{-1}} E_t \left[(r_{t+1} C_{t+1}^{-\eta} + (1-\delta) C_{t+1}^{-\eta}) U_{t+1}^{-\theta^{\text{EZ}}} \right] - C_t^{-\eta} = 0 \quad (5.6)$$

$$-C_t - I_t + Y_t = 0 \quad (5.7)$$

$$I_t - K_t^s + K_{t-1}^s (1-\delta) = 0 \quad (5.8)$$

$$U_t - \beta q_t^{\text{CONSUMER}^1}{}^{(1-\theta^{\text{EZ}})^{-1}} - (-1 + C_t^{1-\eta}) (1-\eta)^{-1} = 0 \quad (5.9)$$

6 Steady state relationships (after reduction)

$$q_{\text{ss}}^{\text{CONSUMER}^1} - U_{\text{ss}}^{1-\theta^{\text{EZ}}} = 0 \quad (6.1)$$

$$-r_{\text{ss}} + \alpha Z_{\text{ss}} 1^{1-\alpha} K_{\text{ss}}^s{}^{-1+\alpha} = 0 \quad (6.2)$$

$$-W_{\text{ss}} + Z_{\text{ss}} (1-\alpha) 1^{-\alpha} K_{\text{ss}}^s{}^\alpha = 0 \quad (6.3)$$

$$-Y_{\text{ss}} + Z_{\text{ss}} 1^{1-\alpha} K_{\text{ss}}^s{}^\alpha = 0 \quad (6.4)$$

$$-Z_{\text{ss}} + e^{\phi \log Z_{\text{ss}}} = 0 \quad (6.5)$$

$$\beta (r_{\text{ss}} C_{\text{ss}}^{-\eta} + (1-\delta) C_{\text{ss}}^{-\eta}) q_{\text{ss}}^{\text{CONSUMER}^1}{}^{-1+(1-\theta^{\text{EZ}})^{-1}} U_{\text{ss}}^{-\theta^{\text{EZ}}} - C_{\text{ss}}^{-\eta} = 0 \quad (6.6)$$

$$-C_{\text{ss}} - I_{\text{ss}} + Y_{\text{ss}} = 0 \quad (6.7)$$

$$I_{\text{ss}} - K_{\text{ss}}^s + K_{\text{ss}}^s (1-\delta) = 0 \quad (6.8)$$

$$U_{\text{ss}} - \beta q_{\text{ss}}^{\text{CONSUMER}^1}{}^{(1-\theta^{\text{EZ}})^{-1}} - (-1 + C_{\text{ss}}^{1-\eta}) (1-\eta)^{-1} = 0 \quad (6.9)$$

7 Calibrating equations

$$-0.36 Y_{\text{ss}} + r_{\text{ss}} K_{\text{ss}}^s = 0 \quad (7.1)$$

8 Parameter settings

$$\beta = 0.99 \quad (8.1)$$

$$\delta = 0.025 \quad (8.2)$$

$$\eta = 2 \quad (8.3)$$

$$\phi = 0.95 \quad (8.4)$$

$$\theta^{\text{EZ}} = 0.05 \quad (8.5)$$

9 Steady-state values

	Steady-state value
q^{CONSUMER^1}	58.4346
r	0.0351
C	3.6213
I	1.4427
K^s	57.7077
U	72.3856
W	3.0384
Y	5.064
Z	1

10 Model parameters

	Value
α	0.4
β	0.99
δ	0.025
η	2
ϕ	0.95
θ^{EZ}	0.05

11 The solution of the 1st order perturbation

Matrix P

$$\begin{matrix} K_{t-1}^s & Z_{t-1} \\ K_t^s \\ Z_t \end{matrix} \begin{pmatrix} 0.9781 & 0.0578 \\ 0 & 0.95 \end{pmatrix}$$

Matrix Q

$$\begin{matrix} \epsilon^Z \\ K^s \\ Z \end{matrix} \begin{pmatrix} 0.0608 \\ 1 \end{pmatrix}$$

Matrix R

$$\begin{matrix} q_t^{\text{CONSUMER}^1} \\ r_t \\ C_t \\ I_t \\ U_t \\ W_t \\ Y_t \end{matrix} \begin{matrix} K_{t-1}^s & Z_{t-1} \\ \begin{pmatrix} 0.0424 & 0.0542 \\ -0.4897 & 0.727 \\ 0.3793 & 0.2084 \\ 0.1223 & 2.3122 \\ 0.0456 & 0.0572 \\ 0.3265 & 0.8616 \\ 0.3061 & 0.8077 \end{pmatrix} \end{matrix}$$

Matrix S

$$\begin{matrix} q^{\text{CONSUMER}^1} \\ r \\ C \\ I \\ U \\ W \\ Y \end{matrix} \begin{matrix} \epsilon^Z \\ \begin{pmatrix} 0.057 \\ 0.7652 \\ 0.2193 \\ 2.4339 \\ 0.0602 \\ 0.9069 \\ 0.8503 \end{pmatrix} \end{matrix}$$

12 Model statistics

12.1 Basic statistics

	Steady-state value	Std. dev.	Variance	Loglin
C	3.6213	0.2991	0.0895	Y
K^s	57.7077	0.2842	0.0807	Y
W	3.0384	1.1799	1.3922	Y
I	1.4427	3.1705	10.052	Y
r	0.0351	1.0158	1.0318	Y
Y	5.064	1.1062	1.2236	Y

12.2 Correlation matrix

	r	C	I	K^s	W	Y
r	1	0.875	0.989	0.082	0.977	0.977
C		1	0.937	0.554	0.958	0.958
I			1	0.228	0.998	0.998
K^s				1	0.293	0.293
W					1	1
Y						1

12.3 Cross correlations with the reference variable (Y)

	$\sigma[\cdot]$ rel. to $\sigma[Y]$	Y_{t-5}	Y_{t-4}	Y_{t-3}	Y_{t-2}	Y_{t-1}	Y_t	Y_{t+1}	Y_{t+2}	Y_{t+3}	Y_{t+4}	Y_{t+5}
r_t	0.918	0.102	0.222	0.368	0.542	0.745	0.977	0.641	0.366	0.147	-0.022	-0.147
C_t	0.27	-0.154	-0.028	0.142	0.359	0.63	0.958	0.769	0.594	0.436	0.297	0.177
I_t	2.866	0.026	0.152	0.309	0.501	0.73	0.998	0.697	0.445	0.239	0.075	-0.052
K_t^s	0.257	-0.493	-0.439	-0.341	-0.193	0.015	0.293	0.48	0.593	0.645	0.649	0.619
W_t	1.067	-0.008	0.119	0.28	0.479	0.718	1	0.718	0.479	0.28	0.119	-0.008
Y_t	1	-0.008	0.119	0.28	0.479	0.718	1	0.718	0.479	0.28	0.119	-0.008

12.4 Autocorrelations

	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5
r	0.713	0.47	0.27	0.109	-0.017
C	0.759	0.544	0.356	0.196	0.063
I	0.714	0.472	0.272	0.111	-0.016
K^s	0.96	0.864	0.731	0.577	0.415
W	0.718	0.479	0.28	0.119	-0.008
Y	0.718	0.479	0.28	0.119	-0.008

13 Impulse response functions

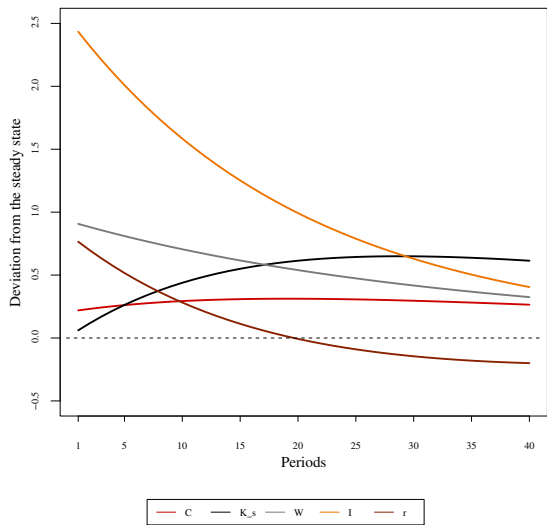


Figure 1: Impulse responses (C, K^s, W, I, r) to ϵ^Z shock

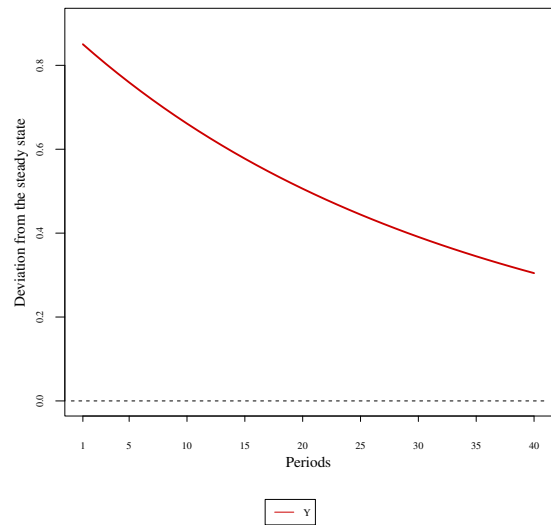


Figure 2: Impulse response (Y) to ϵ^Z shock