

## 1 CONSUMER

### 1.1 Optimisation problem

$$\max_{K_t^s, C_t, I_t} U_t = \beta \left( E_t \left[ U_{t+1}^{1-\theta^{EZ}} \right] \right)^{(1-\theta^{EZ})^{-1}} + (-1 + C_t^{1-\eta}) (1-\eta)^{-1} \quad (1.1)$$

s.t. :

$$C_t + I_t = \pi_t + K_{t-1}^s r_t + L_t^s W_t \quad \left( \lambda_t^{\text{CONSUMER}^1} \right) \quad (1.2)$$

$$K_t^s = I_t + K_{t-1}^s (1-\delta) \quad \left( \lambda_t^{\text{CONSUMER}^2} \right) \quad (1.3)$$

### 1.2 Identities

$$L_t^s = 1 \quad (1.4)$$

### 1.3 First order conditions

$$-\lambda_t^{\text{CONSUMER}^U} + \beta q_{t-1}^{\text{CONSUMER}^1-1+(1-\theta^{EZ})^{-1}} U_t^{-\theta^{EZ}} = 0 \quad (U_t) \quad (1.5)$$

$$-\lambda_t^{\text{CONSUMER}^2} + E_t \left[ \lambda_{t+1}^{\text{CONSUMER}^U} \left( \lambda_{t+1}^{\text{CONSUMER}^1} r_{t+1} + \lambda_{t+1}^{\text{CONSUMER}^2} (1-\delta) \right) \right] = 0 \quad (K_t^s) \quad (1.6)$$

$$-\lambda_t^{\text{CONSUMER}^1} + C_t^{-\eta} = 0 \quad (C_t) \quad (1.7)$$

$$-\lambda_t^{\text{CONSUMER}^1} + \lambda_t^{\text{CONSUMER}^2} = 0 \quad (I_t) \quad (1.8)$$

## 2 FIRM

### 2.1 Optimisation problem

$$\max_{K_t^d, L_t^d, Y_t} \pi_t = Y_t - L_t^d W_t - r_t K_t^d \quad (2.1)$$

s.t. :

$$Y_t = Z_t K_t^{d\alpha} L_t^{d^{1-\alpha}} \quad \left( \lambda_t^{\text{FIRM}^1} \right) \quad (2.2)$$

### 2.2 First order conditions

$$-r_t + \alpha \lambda_t^{\text{FIRM}^1} Z_t K_t^{d-1+\alpha} L_t^{d^{1-\alpha}} = 0 \quad (K_t^d) \quad (2.3)$$

$$-W_t + \lambda_t^{\text{FIRM}^1} Z_t (1-\alpha) K_t^{d\alpha} L_t^{d-\alpha} = 0 \quad (L_t^d) \quad (2.4)$$

$$1 - \lambda_t^{\text{FIRM}^1} = 0 \quad (Y_t) \quad (2.5)$$

### 2.3 First order conditions after reduction

$$-r_t + \alpha Z_t K_t^{d-1+\alpha} L_t^{d^{1-\alpha}} = 0 \quad (K_t^d) \quad (2.6)$$

$$-W_t + Z_t (1-\alpha) K_t^{d\alpha} L_t^{d-\alpha} = 0 \quad (L_t^d) \quad (2.7)$$

### 3 EQUILIBRIUM

#### 3.1 Identities

$$K_t^d = K_{t-1}^s \quad (3.1)$$

$$L_t^d = L_t^s \quad (3.2)$$

### 4 EXOG

#### 4.1 Identities

$$Z_t = e^{\epsilon_t^Z + \phi \log Z_{t-1}} \quad (4.1)$$

### 5 Equilibrium relationships (after reduction)

$$q_t^{\text{CONSUMER}^1} - E_t \left[ U_{t+1}^{1-\theta^{\text{EZ}}} \right] = 0 \quad (5.1)$$

$$-r_t + \alpha Z_t 1^{1-\alpha} K_{t-1}^s^{-1+\alpha} = 0 \quad (5.2)$$

$$-W_t + Z_t (1 - \alpha) 1^{-\alpha} K_{t-1}^s{}^\alpha = 0 \quad (5.3)$$

$$-Y_t + Z_t 1^{1-\alpha} K_{t-1}^s{}^\alpha = 0 \quad (5.4)$$

$$-Z_t + e^{\epsilon_t^Z + \phi \log Z_{t-1}} = 0 \quad (5.5)$$

$$\beta q_t^{\text{CONSUMER}^1 - 1 + (1 - \theta^{\text{EZ}})^{-1}} E_t \left[ (r_{t+1} C_{t+1}^{-\eta} + (1 - \delta) C_{t+1}^{-\eta}) U_{t+1}^{-\theta^{\text{EZ}}} \right] - C_t^{-\eta} = 0 \quad (5.6)$$

$$-C_t - I_t + Y_t = 0 \quad (5.7)$$

$$I_t - K_t^s + K_{t-1}^s (1 - \delta) = 0 \quad (5.8)$$

$$U_t - \beta q_t^{\text{CONSUMER}^1 (1 - \theta^{\text{EZ}})^{-1}} - (-1 + C_t^{1-\eta}) (1 - \eta)^{-1} = 0 \quad (5.9)$$

### 6 Steady state relationships (after reduction)

$$q_{\text{ss}}^{\text{CONSUMER}^1} - U_{\text{ss}}^{1-\theta^{\text{EZ}}} = 0 \quad (6.1)$$

$$-r_{\text{ss}} + \alpha Z_{\text{ss}} 1^{1-\alpha} K_{\text{ss}}^s^{-1+\alpha} = 0 \quad (6.2)$$

$$-W_{\text{ss}} + Z_{\text{ss}} (1 - \alpha) 1^{-\alpha} K_{\text{ss}}^s{}^\alpha = 0 \quad (6.3)$$

$$-Y_{\text{ss}} + Z_{\text{ss}} 1^{1-\alpha} K_{\text{ss}}^s{}^\alpha = 0 \quad (6.4)$$

$$-Z_{\text{ss}} + e^{\phi \log Z_{\text{ss}}} = 0 \quad (6.5)$$

$$\beta (r_{\text{ss}} C_{\text{ss}}^{-\eta} + (1 - \delta) C_{\text{ss}}^{-\eta}) q_{\text{ss}}^{\text{CONSUMER}^1 - 1 + (1 - \theta^{\text{EZ}})^{-1}} U_{\text{ss}}^{-\theta^{\text{EZ}}} - C_{\text{ss}}^{-\eta} = 0 \quad (6.6)$$

$$-C_{\text{ss}} - I_{\text{ss}} + Y_{\text{ss}} = 0 \quad (6.7)$$

$$I_{ss} - K_{ss}^s + K_{ss}^s (1 - \delta) = 0 \quad (6.8)$$

$$U_{ss} - \beta q_{ss}^{\text{CONSUMER}^1} (1 - \theta^{\text{EZ}})^{-1} - (-1 + C_{ss}^{1-\eta}) (1 - \eta)^{-1} = 0 \quad (6.9)$$

## 7 Calibrating equations

$$-0.36Y_{ss} + r_{ss}K_{ss}^s = 0 \quad (7.1)$$

## 8 Parameter settings

$$\beta = 0.99 \quad (8.1)$$

$$\delta = 0.025 \quad (8.2)$$

$$\eta = 2 \quad (8.3)$$

$$\phi = 0.95 \quad (8.4)$$

$$\theta^{\text{EZ}} = 0.05 \quad (8.5)$$

## 9 Steady-state values

	Steady-state value
$q^{\text{CONSUMER}^1}$	58.4346
$r$	0.0351
$C$	3.6213
$I$	1.4427
$K^s$	57.7077
$U$	72.3856
$W$	3.0384
$Y$	5.064
$Z$	1

## 10 Model parameters

	Value
$\alpha$	0.4
$\beta$	0.99
$\delta$	0.025
$\eta$	2
$\phi$	0.95
$\theta^{\text{EZ}}$	0.05

## 11 The solution of the 1st order perturbation

Matrix  $P$

$$K_t^s \begin{pmatrix} K_{t-1}^s & Z_{t-1} \\ 0.9781 & 0.0578 \\ 0 & 0.95 \end{pmatrix}$$

## Matrix $Q$

$$\epsilon^Z \begin{matrix} K^s \\ Z \end{matrix} \begin{pmatrix} 0.0608 \\ 1 \end{pmatrix}$$

## Matrix $R$

$$\begin{matrix} q_t^{\text{CONSUMER}^1} \\ r_t \\ C_t \\ I_t \\ U_t \\ W_t \\ Y_t \end{matrix} \begin{matrix} K_{t-1}^s & Z_{t-1} \\ \begin{pmatrix} 0.0424 & 0.0542 \\ -0.4897 & 0.727 \\ 0.3793 & 0.2084 \\ 0.1223 & 2.3122 \\ 0.0456 & 0.0572 \\ 0.3265 & 0.8616 \\ 0.3061 & 0.8077 \end{pmatrix} \end{matrix}$$

## Matrix $S$

$$\begin{matrix} q^{\text{CONSUMER}^1} \\ r \\ C \\ I \\ U \\ W \\ Y \end{matrix} \begin{matrix} \epsilon^Z \\ \begin{pmatrix} 0.057 \\ 0.7652 \\ 0.2193 \\ 2.4339 \\ 0.0602 \\ 0.9069 \\ 0.8503 \end{pmatrix} \end{matrix}$$

## 12 Model statistics

### 12.1 Basic statistics

	Steady-state value	Std. dev.	Variance	Loglin
$C$	3.6213	0.2991	0.0895	Y
$K^s$	57.7077	0.2842	0.0807	Y
$W$	3.0384	1.1799	1.3922	Y
$I$	1.4427	3.1705	10.052	Y
$r$	0.0351	1.0158	1.0318	Y
$Y$	5.064	1.1062	1.2236	Y

### 12.2 Correlation matrix

	$r$	$C$	$I$	$K^s$	$W$	$Y$
$r$	1	0.875	0.989	0.082	0.977	0.977
$C$		1	0.937	0.554	0.958	0.958
$I$			1	0.228	0.998	0.998
$K^s$				1	0.293	0.293
$W$					1	1
$Y$						1

### 12.3 Cross correlations with the reference variable ( $Y$ )

	$\sigma[\cdot]$ rel. to $\sigma[Y]$	$Y_{t-5}$	$Y_{t-4}$	$Y_{t-3}$	$Y_{t-2}$	$Y_{t-1}$	$Y_t$	$Y_{t+1}$	$Y_{t+2}$	$Y_{t+3}$	$Y_{t+4}$	$Y_{t+5}$
$r_t$	0.918	0.102	0.222	0.368	0.542	0.745	0.977	0.641	0.366	0.147	-0.022	-0.147
$C_t$	0.27	-0.154	-0.028	0.142	0.359	0.63	0.958	0.769	0.594	0.436	0.297	0.177
$I_t$	2.866	0.026	0.152	0.309	0.501	0.73	0.998	0.697	0.445	0.239	0.075	-0.052
$K_t^s$	0.257	-0.493	-0.439	-0.341	-0.193	0.015	0.293	0.48	0.593	0.645	0.649	0.619
$W_t$	1.067	-0.008	0.119	0.28	0.479	0.718	1	0.718	0.479	0.28	0.119	-0.008
$Y_t$	1	-0.008	0.119	0.28	0.479	0.718	1	0.718	0.479	0.28	0.119	-0.008

## 12.4 Autocorrelations

	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5
$r$	0.713	0.47	0.27	0.109	-0.017
$C$	0.759	0.544	0.356	0.196	0.063
$I$	0.714	0.472	0.272	0.111	-0.016
$K^s$	0.96	0.864	0.731	0.577	0.415
$W$	0.718	0.479	0.28	0.119	-0.008
$Y$	0.718	0.479	0.28	0.119	-0.008

# 13 Impulse response functions

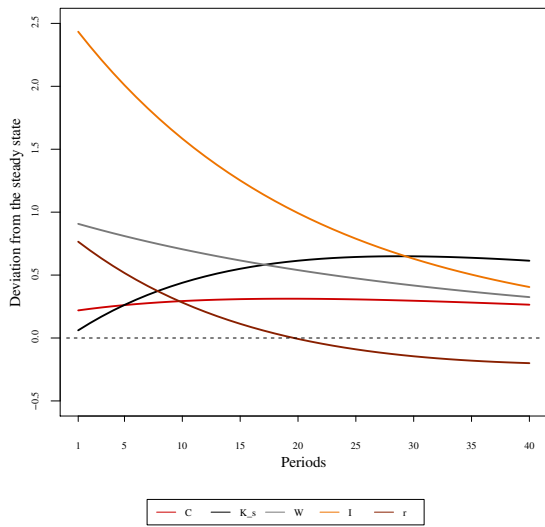


Figure 1: Impulse responses ( $C, K^s, W, I, r$ ) to  $\epsilon^Z$  shock

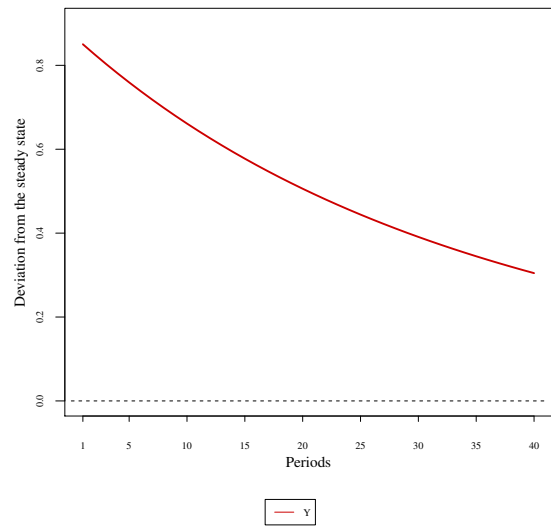


Figure 2: Impulse response ( $Y$ ) to  $\epsilon^Z$  shock