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Index sets

$$IND = \{\mathrm{H}, \mathrm{M}\}$$

1 CONSUMER

1.1 Optimisation problem

$$\max_{\left(K_t^{\langle i \rangle}\right)_{i \in IND}, \left(C_t^{\langle i \rangle}\right)_{i \in IND}, \left(N_t^{\langle i \rangle}\right)_{i \in IND}, \left(I_t^{\langle i \rangle}\right)_{i \in IND}} U_t = \beta \mathrm{E}_t \left[U_{t+1}\right] + \log \left(1 - \sum_{i \in IND} N_t^{\langle i \rangle}\right) (1 - b) + b e^{-1} \log \left(a {C_t^{\langle \mathrm{M} \rangle}}^{e} + (1 - a) {C_t^{\langle \mathrm{H} \rangle}}^{e}\right) \tag{1.1}$$

s.t. :

$$C_t^{\langle \mathrm{M} \rangle} + \sum_{i \in IND} I_t^{\langle i \rangle} = \pi_t + r_t K_{t-1}^{\langle \mathrm{M} \rangle} + W_t N_t^{\langle \mathrm{M} \rangle} \quad \left(\lambda_t^{\mathrm{CONSUMER}^1}\right) \tag{1.2}$$

$$i \in IND: \quad K_t^{\langle i \rangle} = I_t^{\langle i \rangle} + K_{t-1}^{\langle i \rangle} (1 - \delta) \quad \left({\lambda^{\mathrm{CONSUMER}^2}}_t^{\langle i \rangle}\right) \tag{1.3}$$

$$C_t^{\langle \mathrm{H} \rangle} = \Gamma Z_t^{\langle \mathrm{H} \rangle} {K_{t-1}^{\langle \mathrm{H} \rangle}}^{\theta} {N_t^{\langle \mathrm{H} \rangle}}^{1-\theta} \quad \left(\lambda_t^{\mathrm{CONSUMER}^3}\right) \tag{1.4}$$

1.2 Identities

$$K_t = \sum_{i \in IND} K_t^{\langle i \rangle} \tag{1.5}$$

$$I_t = \sum_{i \in IND} I_t^{\langle i \rangle} \tag{1.6}$$

$$N_t = \sum_{i \in IND} N_t^{\langle i \rangle} \tag{1.7}$$

1.3 First order conditions

$$i \in IND: \quad -{\lambda^{\mathrm{CONSUMER}^2}}_t^{\langle i \rangle} + \beta \left(\delta^{\langle \mathrm{M}, i \rangle} \mathrm{E}_t \left[\lambda_{t+1}^{\mathrm{CONSUMER}^1} r_{t+1}\right] + (1 - \delta) \mathrm{E}_t \left[{\lambda^{\mathrm{CONSUMER}^2}}_{t+1}^{\langle i \rangle}\right] + \delta^{\langle \mathrm{H}, i \rangle} \theta \Gamma {K_t^{\langle \mathrm{H} \rangle}}^{-1+\theta} \mathrm{E}_t \left[\lambda_{t+1}^{\mathrm{CONSUMER}^3} Z_{t+1}^{\langle \mathrm{H} \rangle} {N_{t+1}^{\langle \mathrm{H} \rangle}}^{1-\theta}\right]\right) = 0 \quad \left(K_t^{\langle i \rangle}\right) \tag{1.8}$$

$$i \in \mathit{IND}: \quad -\delta^{\langle \mathrm{M},i\rangle}\lambda_t^{\mathrm{CONSUMER}^1} - \delta^{\langle \mathrm{H},i\rangle}\lambda_t^{\mathrm{CONSUMER}^3} + be^{-1}\left(a{C_t^{\langle \mathrm{M}\rangle}}^{e} + (1-a)\,{C_t^{\langle \mathrm{H}\rangle}}^{e}\right)^{-1}\left(\delta^{\langle \mathrm{M},i\rangle} ae{C_t^{\langle \mathrm{M}\rangle}}^{-1+e} + \delta^{\langle \mathrm{H},i\rangle} e\,(1-a)\,{C_t^{\langle \mathrm{H}\rangle}}^{-1+e}\right) = 0 \quad \left(C_t^{\langle i\rangle}\right) \tag{1.9}$$

$$i \in \mathit{IND}: \quad -(1-b)\left(1 - \sum_{i\in \mathit{IND}} N_t^{\langle i\rangle}\right)^{-1} + \delta^{\langle \mathrm{M},i\rangle}\lambda_t^{\mathrm{CONSUMER}^1} W_t + \delta^{\langle \mathrm{H},i\rangle}\Gamma\lambda_t^{\mathrm{CONSUMER}^3} Z_t^{\langle \mathrm{H}\rangle}\,(1-\theta)\,{K_{t-1}^{\langle \mathrm{H}\rangle}}^{\theta}{N_t^{\langle \mathrm{H}\rangle}}^{-\theta} = 0 \quad \left(N_t^{\langle i\rangle}\right) \tag{1.10}$$

$$i \in \mathit{IND}: \quad -\lambda_t^{\mathrm{CONSUMER}^1} + {\lambda^{\mathrm{CONSUMER}^2}}_t^{\langle i\rangle} = 0 \quad \left(I_t^{\langle i\rangle}\right) \tag{1.11}$$

2 FIRM

2.1 Optimisation problem

$$\max_{K_t^{\mathrm{m^d}},N_t^{\mathrm{m^d}},Y_t,\pi_t} \Pi_t = \pi_t \tag{2.1}$$

s.t. :

$$\pi_t = Y_t - N_t^{\mathrm{m^d}} W_t - r_t K_t^{\mathrm{m^d}} \quad \left(\lambda_t^{\mathrm{FIRM}^1}\right) \tag{2.2}$$

$$Y_t = \Gamma Z_t^{\langle \mathrm{M}\rangle}{K_t^{\mathrm{m^d}}}^{\alpha}{N_t^{\mathrm{m^d}}}^{1-\alpha} \quad \left(\lambda_t^{\mathrm{FIRM}^2}\right) \tag{2.3}$$

2.2 First order conditions

$$-\lambda_t^{\mathrm{FIRM}^1} r_t + \alpha\Gamma\lambda_t^{\mathrm{FIRM}^2} Z_t^{\langle \mathrm{M}\rangle}{K_t^{\mathrm{m^d}}}^{-1+\alpha}{N_t^{\mathrm{m^d}}}^{1-\alpha} = 0 \quad \left(K_t^{\mathrm{m^d}}\right) \tag{2.4}$$

$$-\lambda_t^{\mathrm{FIRM}^1} W_t + \Gamma\lambda_t^{\mathrm{FIRM}^2} Z_t^{\langle \mathrm{M}\rangle}\,(1-\alpha)\,{K_t^{\mathrm{m^d}}}^{\alpha}{N_t^{\mathrm{m^d}}}^{-\alpha} = 0 \quad \left(N_t^{\mathrm{m^d}}\right) \tag{2.5}$$

$$\lambda_t^{\mathrm{FIRM}^1} - \lambda_t^{\mathrm{FIRM}^2} = 0 \quad (Y_t) \tag{2.6}$$

$$1 - \lambda_t^{\mathrm{FIRM}^1} = 0 \quad (\pi_t) \tag{2.7}$$

2.3 First order conditions after reduction

$$-r_t + \alpha\Gamma Z_t^{\langle \mathrm{M}\rangle}{K_t^{\mathrm{m^d}}}^{-1+\alpha}{N_t^{\mathrm{m^d}}}^{1-\alpha} = 0 \quad \left(K_t^{\mathrm{m^d}}\right) \tag{2.8}$$

$$-W_t + \Gamma Z_t^{\langle \mathrm{M}\rangle}\,(1-\alpha)\,{K_t^{\mathrm{m^d}}}^{\alpha}{N_t^{\mathrm{m^d}}}^{-\alpha} = 0 \quad \left(N_t^{\mathrm{m^d}}\right) \tag{2.9}$$

3 EQUILIBRIUM

3.1 Identities

$$K_t^{\mathrm{m^d}} = K_{t-1}^{\langle \mathrm{M} \rangle} \tag{3.1}$$

$$N_t^{\mathrm{m^d}} = N_t^{\langle \mathrm{M} \rangle} \tag{3.2}$$

4 EXOG

4.1 Identities

$$i \in \mathit{IND}: \quad Z_t^{\langle i \rangle} = e^{\epsilon_t^{\langle i \rangle} + \psi^{\langle i \rangle} \log Z_{t-1}^{\langle i \rangle}} \tag{4.1}$$

5 Equilibrium relationships (before expansion and reduction)

$$-\pi_t + \Pi_t = 0 \tag{5.1}$$

$$-r_t + \alpha \Gamma Z_t^{\langle \mathrm{M} \rangle} {K_t^{\mathrm{m^d}}}^{-1+\alpha} {N_t^{\mathrm{m^d}}}^{1-\alpha} = 0 \tag{5.2}$$

$$-I_t + \sum_{i \in \mathit{IND}} I_t^{\langle i \rangle} = 0 \tag{5.3}$$

$$-K_t + \sum_{i \in \mathit{IND}} K_t^{\langle i \rangle} = 0 \tag{5.4}$$

$$-K_t^{\mathrm{m^d}} + K_{t-1}^{\langle \mathrm{M} \rangle} = 0 \tag{5.5}$$

$$-N_t + \sum_{i \in \mathit{IND}} N_t^{\langle i \rangle} = 0 \tag{5.6}$$

$$-N_t^{\mathrm{m^d}} + N_t^{\langle \mathrm{M} \rangle} = 0 \tag{5.7}$$

$$-W_t + \Gamma Z_t^{\langle \mathrm{M} \rangle} (1-\alpha) {K_t^{\mathrm{m^d}}}^{\alpha} {N_t^{\mathrm{m^d}}}^{-\alpha} = 0 \tag{5.8}$$

$$-Y_t + \Gamma Z_t^{\langle \mathrm{M} \rangle} {K_t^{\mathrm{m^d}}}^{\alpha} {N_t^{\mathrm{m^d}}}^{1-\alpha} = 0 \tag{5.9}$$

$$-C_t^{\langle \mathrm{H}\rangle} + \Gamma Z_t^{\langle \mathrm{H}\rangle} {K_{t-1}^{\langle \mathrm{H}\rangle}}^{\theta} {N_t^{\langle \mathrm{H}\rangle}}^{1-\theta} = 0 \tag{5.10}$$

$$-\pi_t + Y_t - r_t K_t^{\mathrm{md}} - N_t^{\mathrm{md}} W_t = 0 \tag{5.11}$$

$$U_t - \beta \mathrm{E}_t \left[U_{t+1}\right] - \log \left(1 - \sum_{i \in IND} N_t^{\langle i\rangle}\right) (1-b) - be^{-1} \log \left(a {C_t^{\langle \mathrm{M}\rangle}}^{e} + (1-a) {C_t^{\langle \mathrm{H}\rangle}}^{e}\right) = 0 \tag{5.12}$$

$$\pi_t - C_t^{\langle \mathrm{M}\rangle} + r_t K_{t-1}^{\langle \mathrm{M}\rangle} + W_t N_t^{\langle \mathrm{M}\rangle} - \sum_{i \in IND} I_t^{\langle i\rangle} = 0 \tag{5.13}$$

$$i \in IND\colon \quad -\lambda_t^{\mathrm{CONSUMER}^1} + {\lambda^{\mathrm{CONSUMER}^2}}_t^{\langle i\rangle} = 0 \tag{5.14}$$

$$i \in IND\colon \quad -{\lambda^{\mathrm{CONSUMER}^2}}_t^{\langle i\rangle} + \beta \left(\delta^{\langle \mathrm{M},i\rangle} \mathrm{E}_t \left[\lambda_{t+1}^{\mathrm{CONSUMER}^1} r_{t+1}\right] + (1-\delta) \mathrm{E}_t \left[{\lambda^{\mathrm{CONSUMER}^2}}_{t+1}^{\langle i\rangle}\right] + \delta^{\langle \mathrm{H},i\rangle} \theta \Gamma {K_t^{\langle \mathrm{H}\rangle}}^{-1+\theta} \mathrm{E}_t \left[\lambda_{t+1}^{\mathrm{CONSUMER}^3} Z_{t+1}^{\langle \mathrm{H}\rangle} {N_{t+1}^{\langle \mathrm{H}\rangle}}^{1-\theta}\right]\right) = 0 \tag{5.15}$$

$$i \in IND\colon \quad -Z_t^{\langle i\rangle} + e^{\epsilon_t^{\langle i\rangle} + \psi^{\langle i\rangle} \log Z_{t-1}^{\langle i\rangle}} = 0 \tag{5.16}$$

$$i \in IND\colon \quad I_t^{\langle i\rangle} - K_t^{\langle i\rangle} + K_{t-1}^{\langle i\rangle} (1-\delta) = 0 \tag{5.17}$$

$$i \in IND\colon \quad -\delta^{\langle \mathrm{M},i\rangle} \lambda_t^{\mathrm{CONSUMER}^1} - \delta^{\langle \mathrm{H},i\rangle} \lambda_t^{\mathrm{CONSUMER}^3} + be^{-1} \left(a {C_t^{\langle \mathrm{M}\rangle}}^{e} + (1-a) {C_t^{\langle \mathrm{H}\rangle}}^{e}\right)^{-1} \left(\delta^{\langle \mathrm{M},i\rangle} ae {C_t^{\langle \mathrm{M}\rangle}}^{-1+e} + \delta^{\langle \mathrm{H},i\rangle} e (1-a) {C_t^{\langle \mathrm{H}\rangle}}^{-1+e}\right) = 0 \tag{5.18}$$

$$i \in IND\colon \quad -(1-b) \left(1 - \sum_{i \in IND} N_t^{\langle i\rangle}\right)^{-1} + \delta^{\langle \mathrm{M},i\rangle} \lambda_t^{\mathrm{CONSUMER}^1} W_t + \delta^{\langle \mathrm{H},i\rangle} \Gamma \lambda_t^{\mathrm{CONSUMER}^3} Z_t^{\langle \mathrm{H}\rangle} (1-\theta) {K_{t-1}^{\langle \mathrm{H}\rangle}}^{\theta} {N_t^{\langle \mathrm{H}\rangle}}^{-\theta} = 0 \tag{5.19}$$

6 Equilibrium relationships (after expansion and reduction)

$$-r_t + \alpha \Gamma Z_t^{\langle \mathrm{M}\rangle} {K_{t-1}^{\langle \mathrm{M}\rangle}}^{-1+\alpha} {N_t^{\langle \mathrm{M}\rangle}}^{1-\alpha} = 0 \tag{6.1}$$

$$-W_t + \Gamma Z_t^{\langle \mathrm{M}\rangle} (1-\alpha) {K_{t-1}^{\langle \mathrm{M}\rangle}}^{\alpha} {N_t^{\langle \mathrm{M}\rangle}}^{-\alpha} = 0 \tag{6.2}$$

$$-Y_t + \Gamma Z_t^{\langle \mathrm{M}\rangle} {K_{t-1}^{\langle \mathrm{M}\rangle}}^{\alpha} {N_t^{\langle \mathrm{M}\rangle}}^{1-\alpha} = 0 \tag{6.3}$$

$$-C_t^{\langle \mathrm{H} \rangle} + \Gamma Z_t^{\langle \mathrm{H} \rangle} {K_{t-1}^{\langle \mathrm{H} \rangle}}^{\theta} {N_t^{\langle \mathrm{H} \rangle}}^{1-\theta} = 0 \tag{6.4}$$

$$-Z_t^{\langle \mathrm{H} \rangle} + e^{\epsilon_t^{\langle \mathrm{H} \rangle} + \psi^{\langle \mathrm{H} \rangle} \log Z_{t-1}^{\langle \mathrm{H} \rangle}} = 0 \tag{6.5}$$

$$-Z_t^{\langle \mathrm{M} \rangle} + e^{\epsilon_t^{\langle \mathrm{M} \rangle} + \psi^{\langle \mathrm{M} \rangle} \log Z_{t-1}^{\langle \mathrm{M} \rangle}} = 0 \tag{6.6}$$

$$\beta \left(ab \mathrm{E}_t \left[r_{t+1} \left(a {C_{t+1}^{\langle \mathrm{M} \rangle}}^{e} + (1-a) {C_{t+1}^{\langle \mathrm{H} \rangle}}^{e} \right)^{-1} {C_{t+1}^{\langle \mathrm{M} \rangle}}^{-1+e} \right] + ab (1-\delta) \mathrm{E}_t \left[\left(a {C_{t+1}^{\langle \mathrm{M} \rangle}}^{e} + (1-a) {C_{t+1}^{\langle \mathrm{H} \rangle}}^{e} \right)^{-1} {C_{t+1}^{\langle \mathrm{M} \rangle}}^{-1+e} \right] \right) - ab \left(a {C_t^{\langle \mathrm{M} \rangle}}^{e} + (1-a) {C_t^{\langle \mathrm{H} \rangle}}^{e} \right)^{-1} {C_t^{\langle \mathrm{M} \rangle}}^{-1+e} = 0 \tag{6.7}$$

$$\beta \left(ab (1-\delta) \mathrm{E}_t \left[\left(a {C_{t+1}^{\langle \mathrm{M} \rangle}}^{e} + (1-a) {C_{t+1}^{\langle \mathrm{H} \rangle}}^{e} \right)^{-1} {C_{t+1}^{\langle \mathrm{M} \rangle}}^{-1+e} \right] + b\theta\Gamma (1-a) {K_t^{\langle \mathrm{H} \rangle}}^{-1+\theta} \mathrm{E}_t \left[Z_{t+1}^{\langle \mathrm{H} \rangle} \left(a {C_{t+1}^{\langle \mathrm{M} \rangle}}^{e} + (1-a) {C_{t+1}^{\langle \mathrm{H} \rangle}}^{e} \right)^{-1} {C_{t+1}^{\langle \mathrm{H} \rangle}}^{-1+e} {N_{t+1}^{\langle \mathrm{H} \rangle}}^{1-\theta} \right] \right) - ab \left(a {C_t^{\langle \mathrm{M} \rangle}}^{e} + (1-a) {C_t^{\langle \mathrm{H} \rangle}}^{e} \right)^{-1} C_t^{\langle \mathrm{M} \rangle} \tag{6.8}$$

$$-(1-b) \left(1 - N_t^{\langle \mathrm{H} \rangle} - N_t^{\langle \mathrm{M} \rangle} \right)^{-1} + abW_t \left(a {C_t^{\langle \mathrm{M} \rangle}}^{e} + (1-a) {C_t^{\langle \mathrm{H} \rangle}}^{e} \right)^{-1} {C_t^{\langle \mathrm{M} \rangle}}^{-1+e} = 0 \tag{6.9}$$

$$-(1-b) \left(1 - N_t^{\langle \mathrm{H} \rangle} - N_t^{\langle \mathrm{M} \rangle} \right)^{-1} + b\Gamma Z_t^{\langle \mathrm{H} \rangle} (1-a)(1-\theta) \left(a {C_t^{\langle \mathrm{M} \rangle}}^{e} + (1-a) {C_t^{\langle \mathrm{H} \rangle}}^{e} \right)^{-1} {K_{t-1}^{\langle \mathrm{H} \rangle}}^{\theta} {C_t^{\langle \mathrm{H} \rangle}}^{-1+e} {N_t^{\langle \mathrm{H} \rangle}}^{-\theta} = 0 \tag{6.10}$$

$$-I_t + I_t^{\langle \mathrm{H} \rangle} + I_t^{\langle \mathrm{M} \rangle} = 0 \tag{6.11}$$

$$-K_t + K_t^{\langle \mathrm{H} \rangle} + K_t^{\langle \mathrm{M} \rangle} = 0 \tag{6.12}$$

$$-N_t + N_t^{\langle \mathrm{H} \rangle} + N_t^{\langle \mathrm{M} \rangle} = 0 \tag{6.13}$$

$$I_t^{\langle \mathrm{H} \rangle} - K_t^{\langle \mathrm{H} \rangle} + K_{t-1}^{\langle \mathrm{H} \rangle} (1-\delta) = 0 \tag{6.14}$$

$$I_t^{\langle \mathrm{M} \rangle} - K_t^{\langle \mathrm{M} \rangle} + K_{t-1}^{\langle \mathrm{M} \rangle} (1-\delta) = 0 \tag{6.15}$$

$$U_t - \beta \mathrm{E}_t \left[U_{t+1} \right] - \log \left(1 - N_t^{\langle \mathrm{H} \rangle} - N_t^{\langle \mathrm{M} \rangle} \right) (1-b) - be^{-1} \log \left(a {C_t^{\langle \mathrm{M} \rangle}}^{e} + (1-a) {C_t^{\langle \mathrm{H} \rangle}}^{e} \right) = 0 \tag{6.16}$$

$$Y_t - C_t^{\langle \mathrm{M} \rangle} - I_t^{\langle \mathrm{H} \rangle} - I_t^{\langle \mathrm{M} \rangle} = 0 \tag{6.17}$$

7 Steady state relationships (before expansion and reduction)

$$-\pi_{\mathrm{ss}} + \Pi_{\mathrm{ss}} = 0 \tag{7.1}$$

$$-r_{\mathrm{ss}} + \alpha \Gamma Z_{\mathrm{ss}}^{\langle \mathrm{M} \rangle} {K_{\mathrm{ss}}^{\mathrm{m}^{\mathrm{d}}}}^{-1+\alpha} {N_{\mathrm{ss}}^{\mathrm{m}^{\mathrm{d}}}}^{1-\alpha} = 0 \tag{7.2}$$

$$-I_{\mathrm{ss}} + \sum_{i \in IND} I_{\mathrm{ss}}^{\langle i \rangle} = 0 \tag{7.3}$$

$$-K_{\mathrm{ss}} + \sum_{i \in IND} K_{\mathrm{ss}}^{\langle i \rangle} = 0 \tag{7.4}$$

$$-K_{\mathrm{ss}}^{\mathrm{m}^{\mathrm{d}}} + K_{\mathrm{ss}}^{\langle \mathrm{M} \rangle} = 0 \tag{7.5}$$

$$-N_{\mathrm{ss}} + \sum_{i \in IND} N_{\mathrm{ss}}^{\langle i \rangle} = 0 \tag{7.6}$$

$$-N_{\mathrm{ss}}^{\mathrm{m}^{\mathrm{d}}} + N_{\mathrm{ss}}^{\langle \mathrm{M} \rangle} = 0 \tag{7.7}$$

$$-W_{\mathrm{ss}} + \Gamma Z_{\mathrm{ss}}^{\langle \mathrm{M} \rangle} (1-\alpha) {K_{\mathrm{ss}}^{\mathrm{m}^{\mathrm{d}}}}^{\alpha} {N_{\mathrm{ss}}^{\mathrm{m}^{\mathrm{d}}}}^{-\alpha} = 0 \tag{7.8}$$

$$-Y_{\mathrm{ss}} + \Gamma Z_{\mathrm{ss}}^{\langle \mathrm{M} \rangle} {K_{\mathrm{ss}}^{\mathrm{m}^{\mathrm{d}}}}^{\alpha} {N_{\mathrm{ss}}^{\mathrm{m}^{\mathrm{d}}}}^{1-\alpha} = 0 \tag{7.9}$$

$$-C_{\mathrm{ss}}^{\langle \mathrm{H} \rangle} + \Gamma Z_{\mathrm{ss}}^{\langle \mathrm{H} \rangle} {K_{\mathrm{ss}}^{\langle \mathrm{H} \rangle}}^{\theta} {N_{\mathrm{ss}}^{\langle \mathrm{H} \rangle}}^{1-\theta} = 0 \tag{7.10}$$

$$-\pi_{\mathrm{ss}} + Y_{\mathrm{ss}} - r_{\mathrm{ss}} K_{\mathrm{ss}}^{\mathrm{m}^{\mathrm{d}}} - N_{\mathrm{ss}}^{\mathrm{m}^{\mathrm{d}}} W_{\mathrm{ss}} = 0 \tag{7.11}$$

$$U_{\mathrm{ss}} - \beta U_{\mathrm{ss}} - \log\left(1 - \sum_{i \in IND} N_{\mathrm{ss}}^{\langle i \rangle}\right)(1-b) - b e^{-1} \log\left(a {C_{\mathrm{ss}}^{\langle \mathrm{M} \rangle}}^{e} + (1-a) {C_{\mathrm{ss}}^{\langle \mathrm{H} \rangle}}^{e}\right) = 0 \tag{7.12}$$

$$\pi_{\mathrm{ss}} - C_{\mathrm{ss}}^{\langle \mathrm{M} \rangle} + r_{\mathrm{ss}} K_{\mathrm{ss}}^{\langle \mathrm{M} \rangle} + W_{\mathrm{ss}} N_{\mathrm{ss}}^{\langle \mathrm{M} \rangle} - \sum_{i \in IND} I_{\mathrm{ss}}^{\langle i \rangle} = 0 \tag{7.13}$$

$$i \in IND\colon \quad -\lambda_{\mathrm{ss}}^{\mathrm{CONSUMER}^1} + {\lambda^{\mathrm{CONSUMER}^2}}_{\mathrm{ss}}^{\langle i \rangle} = 0 \tag{7.14}$$

$$i \in IND\colon \quad -{\lambda^{\mathrm{CONSUMER}^2}}_{\mathrm{ss}}^{\langle i \rangle} + \beta\left({\lambda^{\mathrm{CONSUMER}^2}}_{\mathrm{ss}}^{\langle i \rangle}(1-\delta) + \delta^{\langle \mathrm{M},i \rangle} \lambda_{\mathrm{ss}}^{\mathrm{CONSUMER}^1} r_{\mathrm{ss}} + \delta^{\langle \mathrm{H},i \rangle} \theta \Gamma \lambda_{\mathrm{ss}}^{\mathrm{CONSUMER}^3} Z_{\mathrm{ss}}^{\langle \mathrm{H} \rangle} {K_{\mathrm{ss}}^{\langle \mathrm{H} \rangle}}^{-1+\theta} {N_{\mathrm{ss}}^{\langle \mathrm{H} \rangle}}^{1-\theta}\right) = 0 \tag{7.15}$$

$$i \in \mathit{IND}\colon\quad -Z_{\mathrm{ss}}^{\langle i\rangle} + e^{\epsilon_{\mathrm{ss}}^{\langle i\rangle} + \psi^{\langle i\rangle}\log Z_{\mathrm{ss}}^{\langle i\rangle}} = 0 \tag{7.16}$$

$$i \in \mathit{IND}\colon\quad I_{\mathrm{ss}}^{\langle i\rangle} - K_{\mathrm{ss}}^{\langle i\rangle} + K_{\mathrm{ss}}^{\langle i\rangle}\left(1-\delta\right) = 0 \tag{7.17}$$

$$i \in \mathit{IND}\colon\quad -\delta^{\langle \mathrm{M},i\rangle}\lambda_{\mathrm{ss}}^{\mathrm{CONSUMER}^1} - \delta^{\langle \mathrm{H},i\rangle}\lambda_{\mathrm{ss}}^{\mathrm{CONSUMER}^3} + be^{-1}\left(aC_{\mathrm{ss}}^{\langle \mathrm{M}\rangle\,e} + \left(1-a\right)C_{\mathrm{ss}}^{\langle \mathrm{H}\rangle\,e}\right)^{-1}\left(\delta^{\langle \mathrm{M},i\rangle}aeC_{\mathrm{ss}}^{\langle \mathrm{M}\rangle\,-1+e} + \delta^{\langle \mathrm{H},i\rangle}e\left(1-a\right)C_{\mathrm{ss}}^{\langle \mathrm{H}\rangle\,-1+e}\right) = 0 \tag{7.18}$$

$$i \in \mathit{IND}\colon\quad -\left(1-b\right)\left(1-\sum_{i\in \mathit{IND}} N_{\mathrm{ss}}^{\langle i\rangle}\right)^{-1} + \delta^{\langle \mathrm{M},i\rangle}\lambda_{\mathrm{ss}}^{\mathrm{CONSUMER}^1}W_{\mathrm{ss}} + \delta^{\langle \mathrm{H},i\rangle}\Gamma\lambda_{\mathrm{ss}}^{\mathrm{CONSUMER}^3}Z_{\mathrm{ss}}^{\langle \mathrm{H}\rangle}\left(1-\theta\right)K_{\mathrm{ss}}^{\langle \mathrm{H}\rangle\,\theta}N_{\mathrm{ss}}^{\langle \mathrm{H}\rangle\,-\theta} = 0 \tag{7.19}$$

8 Steady state relationships (after expansion and reduction)

$$-r_{\mathrm{ss}} + \alpha\Gamma Z_{\mathrm{ss}}^{\langle \mathrm{M}\rangle}K_{\mathrm{ss}}^{\langle \mathrm{M}\rangle\,-1+\alpha}N_{\mathrm{ss}}^{\langle \mathrm{M}\rangle\,1-\alpha} = 0 \tag{8.1}$$

$$-W_{\mathrm{ss}} + \Gamma Z_{\mathrm{ss}}^{\langle \mathrm{M}\rangle}\left(1-\alpha\right)K_{\mathrm{ss}}^{\langle \mathrm{M}\rangle\,\alpha}N_{\mathrm{ss}}^{\langle \mathrm{M}\rangle\,-\alpha} = 0 \tag{8.2}$$

$$-Y_{\mathrm{ss}} + \Gamma Z_{\mathrm{ss}}^{\langle \mathrm{M}\rangle}K_{\mathrm{ss}}^{\langle \mathrm{M}\rangle\,\alpha}N_{\mathrm{ss}}^{\langle \mathrm{M}\rangle\,1-\alpha} = 0 \tag{8.3}$$

$$-C_{\mathrm{ss}}^{\langle \mathrm{H}\rangle} + \Gamma Z_{\mathrm{ss}}^{\langle \mathrm{H}\rangle}K_{\mathrm{ss}}^{\langle \mathrm{H}\rangle\,\theta}N_{\mathrm{ss}}^{\langle \mathrm{H}\rangle\,1-\theta} = 0 \tag{8.4}$$

$$-Z_{\mathrm{ss}}^{\langle \mathrm{H}\rangle} + e^{\psi^{\langle \mathrm{H}\rangle}\log Z_{\mathrm{ss}}^{\langle \mathrm{H}\rangle}} = 0 \tag{8.5}$$

$$-Z_{\mathrm{ss}}^{\langle \mathrm{M}\rangle} + e^{\psi^{\langle \mathrm{M}\rangle}\log Z_{\mathrm{ss}}^{\langle \mathrm{M}\rangle}} = 0 \tag{8.6}$$

$$\beta\left(abr_{\mathrm{ss}}\left(aC_{\mathrm{ss}}^{\langle \mathrm{M}\rangle\,e} + \left(1-a\right)C_{\mathrm{ss}}^{\langle \mathrm{H}\rangle\,e}\right)^{-1}C_{\mathrm{ss}}^{\langle \mathrm{M}\rangle\,-1+e} + ab\left(1-\delta\right)\left(aC_{\mathrm{ss}}^{\langle \mathrm{M}\rangle\,e} + \left(1-a\right)C_{\mathrm{ss}}^{\langle \mathrm{H}\rangle\,e}\right)^{-1}C_{\mathrm{ss}}^{\langle \mathrm{M}\rangle\,-1+e}\right) - ab\left(aC_{\mathrm{ss}}^{\langle \mathrm{M}\rangle\,e} + \left(1-a\right)C_{\mathrm{ss}}^{\langle \mathrm{H}\rangle\,e}\right)^{-1}C_{\mathrm{ss}}^{\langle \mathrm{M}\rangle\,-1+e} = 0 \tag{8.7}$$

$$\beta\left(ab\left(1-\delta\right)\left(aC_{\mathrm{ss}}^{\langle \mathrm{M}\rangle\,e} + \left(1-a\right)C_{\mathrm{ss}}^{\langle \mathrm{H}\rangle\,e}\right)^{-1}C_{\mathrm{ss}}^{\langle \mathrm{M}\rangle\,-1+e} + b\theta\Gamma Z_{\mathrm{ss}}^{\langle \mathrm{H}\rangle}\left(1-a\right)\left(aC_{\mathrm{ss}}^{\langle \mathrm{M}\rangle\,e} + \left(1-a\right)C_{\mathrm{ss}}^{\langle \mathrm{H}\rangle\,e}\right)^{-1}C_{\mathrm{ss}}^{\langle \mathrm{H}\rangle\,-1+e}K_{\mathrm{ss}}^{\langle \mathrm{H}\rangle\,-1+\theta}N_{\mathrm{ss}}^{\langle \mathrm{H}\rangle\,1-\theta}\right) - ab\left(aC_{\mathrm{ss}}^{\langle \mathrm{M}\rangle\,e} + \left(1-a\right)C_{\mathrm{ss}}^{\langle \mathrm{H}\rangle\,e}\right)^{-1}C_{\mathrm{ss}}^{\langle \mathrm{M}\rangle\,-1+e} = 0 \tag{8.8}$$

$$-\left(1-b\right)\left(1-N_{\mathrm{ss}}^{\langle \mathrm{H}\rangle} - N_{\mathrm{ss}}^{\langle \mathrm{M}\rangle}\right)^{-1} + abW_{\mathrm{ss}}\left(aC_{\mathrm{ss}}^{\langle \mathrm{M}\rangle\,e} + \left(1-a\right)C_{\mathrm{ss}}^{\langle \mathrm{H}\rangle\,e}\right)^{-1}C_{\mathrm{ss}}^{\langle \mathrm{M}\rangle\,-1+e} = 0 \tag{8.9}$$

$$-\left(1-b\right)\left(1-N_{\mathrm{ss}}^{\langle\mathrm{H}\rangle}-N_{\mathrm{ss}}^{\langle\mathrm{M}\rangle}\right)^{-1}+b\Gamma Z_{\mathrm{ss}}^{\langle\mathrm{H}\rangle}\left(1-a\right)\left(1-\theta\right)\left(aC_{\mathrm{ss}}^{\langle\mathrm{M}\rangle\,e}+\left(1-a\right)C_{\mathrm{ss}}^{\langle\mathrm{H}\rangle\,e}\right)^{-1}C_{\mathrm{ss}}^{\langle\mathrm{H}\rangle\,-1+e}K_{\mathrm{ss}}^{\langle\mathrm{H}\rangle\,\theta}N_{\mathrm{ss}}^{\langle\mathrm{H}\rangle\,-\theta}=0 \tag{8.10}$$

$$-I_{\mathrm{ss}}+I_{\mathrm{ss}}^{\langle\mathrm{H}\rangle}+I_{\mathrm{ss}}^{\langle\mathrm{M}\rangle}=0 \tag{8.11}$$

$$-K_{\mathrm{ss}}+K_{\mathrm{ss}}^{\langle\mathrm{H}\rangle}+K_{\mathrm{ss}}^{\langle\mathrm{M}\rangle}=0 \tag{8.12}$$

$$-N_{\mathrm{ss}}+N_{\mathrm{ss}}^{\langle\mathrm{H}\rangle}+N_{\mathrm{ss}}^{\langle\mathrm{M}\rangle}=0 \tag{8.13}$$

$$I_{\mathrm{ss}}^{\langle\mathrm{H}\rangle}-K_{\mathrm{ss}}^{\langle\mathrm{H}\rangle}+K_{\mathrm{ss}}^{\langle\mathrm{H}\rangle}\left(1-\delta\right)=0 \tag{8.14}$$

$$I_{\mathrm{ss}}^{\langle\mathrm{M}\rangle}-K_{\mathrm{ss}}^{\langle\mathrm{M}\rangle}+K_{\mathrm{ss}}^{\langle\mathrm{M}\rangle}\left(1-\delta\right)=0 \tag{8.15}$$

$$U_{\mathrm{ss}}-\beta U_{\mathrm{ss}}-\log\left(1-N_{\mathrm{ss}}^{\langle\mathrm{H}\rangle}-N_{\mathrm{ss}}^{\langle\mathrm{M}\rangle}\right)\left(1-b\right)-be^{-1}\log\left(aC_{\mathrm{ss}}^{\langle\mathrm{M}\rangle\,e}+\left(1-a\right)C_{\mathrm{ss}}^{\langle\mathrm{H}\rangle\,e}\right)=0 \tag{8.16}$$

$$Y_{\mathrm{ss}}-C_{\mathrm{ss}}^{\langle\mathrm{M}\rangle}-I_{\mathrm{ss}}^{\langle\mathrm{H}\rangle}-I_{\mathrm{ss}}^{\langle\mathrm{M}\rangle}=0 \tag{8.17}$$

∞

9 Parameter settings

$$a=0.337 \tag{9.1}$$

$$\alpha=0.36 \tag{9.2}$$

$$b=0.63 \tag{9.3}$$

$$\beta=0.99 \tag{9.4}$$

$$\delta=0.025 \tag{9.5}$$

$$e=0.8 \tag{9.6}$$

$$\theta=0.08 \tag{9.7}$$

$$\Gamma = 1 \tag{9.8}$$

$$\psi^{\langle \mathrm{H} \rangle} = 0.95 \tag{9.9}$$

$$\psi^{\langle \mathrm{M} \rangle} = 0.95 \tag{9.10}$$

10 Steady-state values

	Steady-state value
r	0.0351
I	0.3143
K	12.5726
N	0.6102
U	-79.6929
W	2.3706
Y	1.0367
$C^{\langle \mathrm{H} \rangle}$	0.3805
$C^{\langle \mathrm{M} \rangle}$	0.7224
$I^{\langle \mathrm{H} \rangle}$	0.0485
$I^{\langle \mathrm{M} \rangle}$	0.2658
$K^{\langle \mathrm{H} \rangle}$	1.9397
$K^{\langle \mathrm{M} \rangle}$	10.6329
$N^{\langle \mathrm{H} \rangle}$	0.3303
$N^{\langle \mathrm{M} \rangle}$	0.2799
$Z^{\langle \mathrm{H} \rangle}$	1
$Z^{\langle \mathrm{M} \rangle}$	1

11 The solution of the 1st order perturbation

Matrix P

$$
\begin{array}{c} \\ K_t^{\langle \mathrm{H} \rangle} \\ K_t^{\langle \mathrm{M} \rangle} \\ Z_t^{\langle \mathrm{H} \rangle} \\ Z_t^{\langle \mathrm{M} \rangle} \end{array}
\begin{array}{c}
\begin{array}{cccc} K_{t-1}^{\langle \mathrm{H} \rangle} & K_{t-1}^{\langle \mathrm{M} \rangle} & Z_{t-1}^{\langle \mathrm{H} \rangle} & Z_{t-1}^{\langle \mathrm{M} \rangle} \end{array} \\
\begin{pmatrix}
0.0826 & 0.4683 & 2.0323 & -2.6403 \\
0.1545 & 0.8762 & -0.3729 & 0.6255 \\
0 & 0 & 0.95 & 0 \\
0 & 0 & 0 & 0.95
\end{pmatrix}
\end{array}
$$

Matrix Q

$$
\begin{array}{c|cc}
 & \epsilon^{\langle \mathrm{H} \rangle} & \epsilon^{\langle \mathrm{M} \rangle} \\
K^{\langle \mathrm{H} \rangle} & 2.1393 & -2.7792 \\
K^{\langle \mathrm{M} \rangle} & -0.3926 & 0.6584 \\
Z^{\langle \mathrm{H} \rangle} & 1 & 0 \\
Z^{\langle \mathrm{M} \rangle} & 0 & 1
\end{array}
$$

Matrix R

$$
\begin{array}{c|cccc}
 & K_{t-1}^{\langle \mathrm{H} \rangle} & K_{t-1}^{\langle \mathrm{M} \rangle} & Z_{t-1}^{\langle \mathrm{H} \rangle} & Z_{t-1}^{\langle \mathrm{M} \rangle} \\
r_t & -0.08 & -0.4894 & -0.6218 & 1.96 \\
I_t & -0.2798 & -0.4533 & -0.0746 & 4.867 \\
K_t & 0.1434 & 0.8132 & -0.0019 & 0.1217 \\
N_t & -0.0155 & -0.0751 & 0.0429 & 0.226 \\
U_t & 0.0098 & 0.054 & 0.0683 & 0.0832 \\
W_t & 0.045 & 0.2753 & 0.3497 & 0.3819 \\
Y_t & -0.08 & 0.5106 & -0.6218 & 1.96 \\
C_t^{\langle \mathrm{H} \rangle} & 0.1511 & -0.3112 & 1.7804 & -0.8463 \\
C_t^{\langle \mathrm{M} \rangle} & 0.0069 & 0.93 & -0.8599 & 0.6952 \\
I_t^{\langle \mathrm{H} \rangle} & -35.696 & 18.734 & 81.2939 & -105.6101 \\
I_t^{\langle \mathrm{M} \rangle} & 6.1809 & -3.9534 & -14.918 & 25.0205 \\
N_t^{\langle \mathrm{H} \rangle} & 0.0772 & -0.3382 & 0.9026 & -0.9199 \\
N_t^{\langle \mathrm{M} \rangle} & -0.125 & 0.2353 & -0.9715 & 1.5781
\end{array}
$$

Matrix S

$$
\begin{array}{l}
 \\
r \\
I \\
K \\
N \\
U \\
W \\
Y \\
C^{\langle \mathrm{H} \rangle} \\
C^{\langle \mathrm{M} \rangle} \\
I^{\langle \mathrm{H} \rangle} \\
I^{\langle \mathrm{M} \rangle} \\
N^{\langle \mathrm{H} \rangle} \\
N^{\langle \mathrm{M} \rangle}
\end{array}
\begin{array}{c}
\begin{array}{cc} \epsilon^{\langle \mathrm{H} \rangle} & \epsilon^{\langle \mathrm{M} \rangle} \end{array} \\
\begin{pmatrix}
-0.6545 & 2.0631 \\
-0.0785 & 5.1231 \\
-0.002 & 0.1281 \\
0.0452 & 0.2379 \\
0.0719 & 0.0875 \\
0.3682 & 0.402 \\
-0.6545 & 2.0631 \\
1.8741 & -0.8908 \\
-0.9051 & 0.7318 \\
85.5725 & -111.1686 \\
-15.7031 & 26.3373 \\
0.9501 & -0.9683 \\
-1.0227 & 1.6612
\end{pmatrix}
\end{array}
$$

12 Model statistics

12.1 Basic statistics

	Steady-state value	Std. dev.	Variance	Loglin
$C^{\langle M \rangle}$	0.7224	0.9767	0.954	Y
$C^{\langle H \rangle}$	0.3805	1.52	2.3104	Y
Y	1.0367	1.7868	3.1926	Y
$I^{\langle M \rangle}$	0.2658	12.888	166.0993	Y
$I^{\langle H \rangle}$	0.0485	57.2629	3279.0405	Y
$K^{\langle M \rangle}$	10.6329	0.6337	0.4016	Y
$K^{\langle H \rangle}$	1.9397	1.9051	3.6295	Y
$N^{\langle M \rangle}$	0.2799	1.3761	1.8936	Y
$N^{\langle H \rangle}$	0.3303	0.9286	0.8623	Y
W	2.3706	0.6601	0.4358	Y

12.2 Correlation matrix

	W	Y	$C^{\langle H \rangle}$	$C^{\langle M \rangle}$	$I^{\langle H \rangle}$	$I^{\langle M \rangle}$	$K^{\langle H \rangle}$	$K^{\langle M \rangle}$	$N^{\langle H \rangle}$	$N^{\langle M \rangle}$
W	1	0.735	0.421	0.056	-0.028	0.409	-0.271	0.511	-0.148	0.475
Y		1	-0.288	0.622	-0.16	0.542	-0.829	0.779	-0.768	0.946
$C^{\langle H \rangle}$			1	-0.843	0.084	-0.05	0.657	-0.438	0.833	-0.575
$C^{\langle M \rangle}$				1	0.021	0.114	-0.704	0.842	-0.933	0.781
$I^{\langle H \rangle}$					1	-0.901	0.409	-0.068	0.135	-0.194
$I^{\langle M \rangle}$						1	-0.634	0.303	-0.332	0.508
$K^{\langle H \rangle}$							1	-0.592	0.906	-0.946
$K^{\langle M \rangle}$								1	-0.76	0.766
$N^{\langle H \rangle}$									1	-0.927
$N^{\langle M \rangle}$										1

12.3 Cross correlations with the reference variable (Y)

	$\sigma[\cdot]$ rel. to $\sigma[Y]$	Y_{t-5}	Y_{t-4}	Y_{t-3}	Y_{t-2}	Y_{t-1}	Y_t	Y_{t+1}	Y_{t+2}	Y_{t+3}	Y_{t+4}	Y_{t+5}
W_t	0.369	-0.032	0.068	0.197	0.356	0.547	0.735	0.568	0.415	0.281	0.166	0.071
Y_t	1	0.008	0.146	0.32	0.533	0.787	1	0.787	0.533	0.32	0.146	0.008
$C_t^{\langle \mathrm{H} \rangle}$	0.851	0.044	-0.001	-0.06	-0.136	-0.229	-0.288	-0.318	-0.238	-0.168	-0.108	-0.057
$C_t^{\langle \mathrm{M} \rangle}$	0.547	-0.194	-0.098	0.033	0.204	0.42	0.622	0.68	0.584	0.483	0.383	0.287
$I_t^{\langle \mathrm{H} \rangle}$	32.048	-0.124	-0.156	-0.192	-0.23	-0.27	-0.16	0.372	0.307	0.246	0.192	0.144
$I_t^{\langle \mathrm{M} \rangle}$	7.213	0.145	0.226	0.322	0.433	0.558	0.542	-0.016	-0.08	-0.126	-0.155	-0.172
$K_t^{\langle \mathrm{H} \rangle}$	1.066	-0.12	-0.235	-0.373	-0.537	-0.727	-0.829	-0.528	-0.284	-0.092	0.054	0.161
$K_t^{\langle \mathrm{M} \rangle}$	0.355	-0.27	-0.149	0.019	0.238	0.516	0.779	0.751	0.692	0.61	0.516	0.416
$N_t^{\langle \mathrm{H} \rangle}$	0.52	0.04	-0.07	-0.212	-0.387	-0.6	-0.768	-0.677	-0.484	-0.318	-0.179	-0.065
$N_t^{\langle \mathrm{M} \rangle}$	0.77	0.026	0.157	0.322	0.522	0.76	0.946	0.75	0.493	0.281	0.11	-0.024

12.4 Autocorrelations

	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5
W	0.74	0.511	0.317	0.156	0.026
Y	0.787	0.533	0.32	0.146	0.008
$C^{\langle \mathrm{H} \rangle}$	0.771	0.512	0.298	0.124	-0.011
$C^{\langle \mathrm{M} \rangle}$	0.852	0.622	0.418	0.241	0.092
$I^{\langle \mathrm{H} \rangle}$	-0.078	-0.074	-0.068	-0.061	-0.054
$I^{\langle \mathrm{M} \rangle}$	0.076	0.032	-0.003	-0.029	-0.048
$K^{\langle \mathrm{H} \rangle}$	0.711	0.467	0.266	0.105	-0.021
$K^{\langle \mathrm{M} \rangle}$	0.868	0.716	0.556	0.398	0.247
$N^{\langle \mathrm{H} \rangle}$	0.82	0.555	0.333	0.152	0.008
$N^{\langle \mathrm{M} \rangle}$	0.803	0.537	0.316	0.136	-0.005

12.5 Variance decomposition

	$\epsilon^{\langle \mathrm{H} \rangle}$	$\epsilon^{\langle \mathrm{M} \rangle}$
W	0.796	0.204
Y	0.136	0.864
$C^{\langle \mathrm{H} \rangle}$	0.616	0.384
$C^{\langle \mathrm{M} \rangle}$	0.238	0.762
$I^{\langle \mathrm{H} \rangle}$	0.015	0.985
$I^{\langle \mathrm{M} \rangle}$	0.053	0.947
$K^{\langle \mathrm{H} \rangle}$	0.023	0.977
$K^{\langle \mathrm{M} \rangle}$	0.198	0.802
$N^{\langle \mathrm{H} \rangle}$	0.106	0.894
$N^{\langle \mathrm{M} \rangle}$	0.003	0.997

13 Impulse response functions

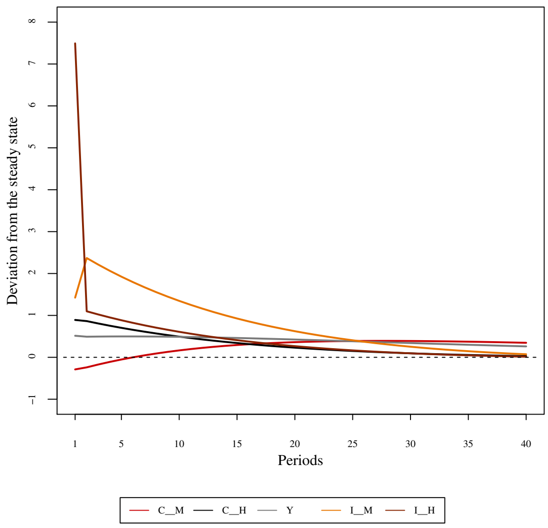

Figure 1: Impulse responses $(C^{\langle \mathrm{M} \rangle}, C^{\langle \mathrm{H} \rangle}, Y, I^{\langle \mathrm{M} \rangle}, I^{\langle \mathrm{H} \rangle})$ to $\epsilon^{\langle \mathrm{H} \rangle}$ shock

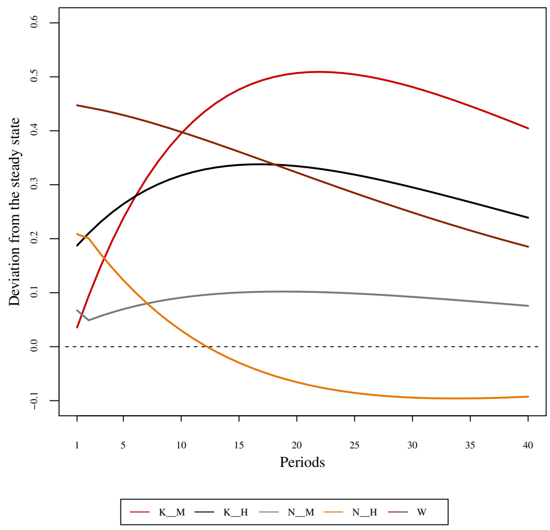

Figure 2: Impulse responses $(K^{\langle \mathrm{M} \rangle}, K^{\langle \mathrm{H} \rangle}, N^{\langle \mathrm{M} \rangle}, N^{\langle \mathrm{H} \rangle}, W)$ to $\epsilon^{\langle \mathrm{H} \rangle}$ shock

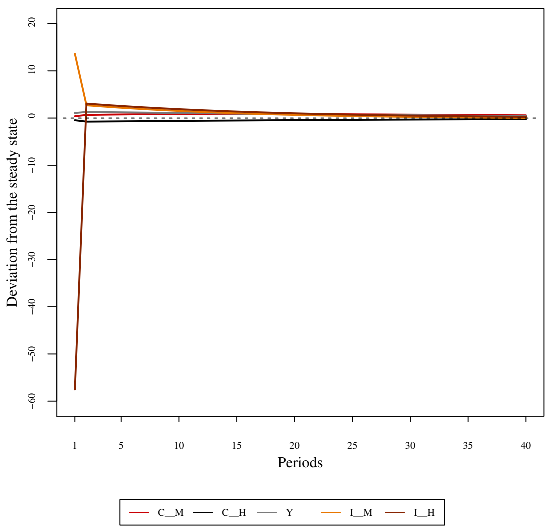

Figure 3: Impulse responses $(C^{\langle \mathrm{M} \rangle}, C^{\langle \mathrm{H} \rangle}, Y, I^{\langle \mathrm{M} \rangle}, I^{\langle \mathrm{H} \rangle})$ to $\epsilon^{\langle \mathrm{M} \rangle}$ shock

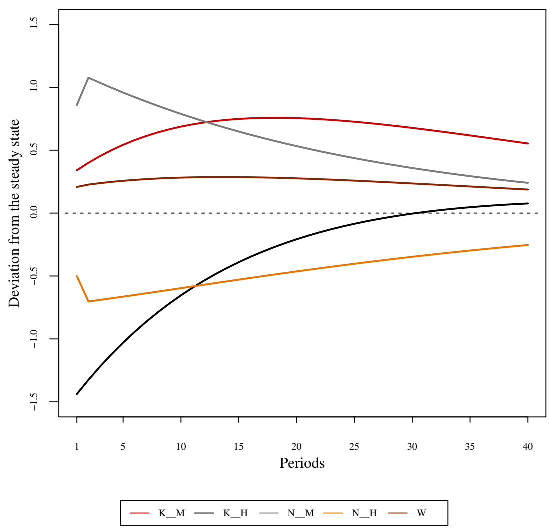

Figure 4: Impulse responses $(K^{\langle \mathrm{M} \rangle}, K^{\langle \mathrm{H} \rangle}, N^{\langle \mathrm{M} \rangle}, N^{\langle \mathrm{H} \rangle}, W)$ to $\epsilon^{\langle \mathrm{M} \rangle}$ shock