

Index sets

$$IND = \{H, M\}$$

1 CONSUMER

1.1 Optimisation problem

$$\max_{(K_t^{(i)})_{i \in IND}, (C_t^{(i)})_{i \in IND}, (N_t^{(i)})_{i \in IND}, (I_t^{(i)})_{i \in IND}} U_t = \beta E_t [U_{t+1}] + \log \left(1 - \sum_{i \in IND} N_t^{(i)} \right) (1 - b) + b e^{-1} \log \left(a C_t^{(M)e} + (1 - a) C_t^{(H)e} \right) \quad (1.1)$$

s.t. :

$$C_t^{(M)} + \sum_{i \in IND} I_t^{(i)} = \pi_t + r_t K_{t-1}^{(M)} + W_t N_t^{(M)} \quad \left(\lambda_t^{\text{CONSUMER}^1} \right) \quad (1.2)$$

$$i \in IND: \quad K_t^{(i)} = I_t^{(i)} + K_{t-1}^{(i)} (1 - \delta) \quad \left(\lambda_t^{\text{CONSUMER}^2(i)} \right) \quad (1.3)$$

$$C_t^{(H)} = \Gamma Z_t^{(H)} K_{t-1}^{(H)\theta} N_t^{(H)1-\theta} \quad \left(\lambda_t^{\text{CONSUMER}^3} \right) \quad (1.4)$$

1.2 Identities

$$K_t = \sum_{i \in IND} K_t^{(i)} \quad (1.5)$$

$$I_t = \sum_{i \in IND} I_t^{(i)} \quad (1.6)$$

$$N_t = \sum_{i \in IND} N_t^{(i)} \quad (1.7)$$

1.3 First order conditions

$$i \in IND: \quad -\lambda^{\text{CONSUMER}^2(i)} + \beta \left(\delta^{(M,i)} E_t \left[\lambda_{t+1}^{\text{CONSUMER}^1} r_{t+1} \right] + (1 - \delta) E_t \left[\lambda_{t+1}^{\text{CONSUMER}^2(i)} + \delta^{(H,i)} \theta \Gamma K_t^{(H)-1+\theta} E_t \left[\lambda_{t+1}^{\text{CONSUMER}^3} Z_{t+1}^{(H)} N_{t+1}^{(H)1-\theta} \right] \right] \right) = 0 \quad \left(K_t^{(i)} \right) \quad (1.8)$$

$$i \in IND: \quad -\delta^{(M,i)} \lambda_t^{\text{CONSUMER}^1} - \delta^{(H,i)} \lambda_t^{\text{CONSUMER}^3} + b e^{-1} \left(a C_t^{(M)e} + (1 - a) C_t^{(H)e} \right)^{-1} \left(\delta^{(M,i)} a e C_t^{(M)-1+e} + \delta^{(H,i)} e (1 - a) C_t^{(H)-1+e} \right) = 0 \quad \left(C_t^{(i)} \right) \quad (1.9)$$

$$i \in IND: \quad -(1-b) \left(1 - \sum_{i \in IND} N_t^{(i)}\right)^{-1} + \delta^{(M,i)} \lambda_t^{\text{CONSUMER}^1} W_t + \delta^{(H,i)} \Gamma \lambda_t^{\text{CONSUMER}^3} Z_t^{(H)} (1-\theta) K_{t-1}^{(H)\theta} N_t^{(H)-\theta} = 0 \quad \left(N_t^{(i)}\right) \quad (1.10)$$

$$i \in IND: \quad -\lambda_t^{\text{CONSUMER}^1} + \lambda_t^{\text{CONSUMER}^2 \langle i \rangle} = 0 \quad \left(I_t^{(i)}\right) \quad (1.11)$$

2 FIRM

2.1 Optimisation problem

$$\max_{K_t^{\text{m}^d}, N_t^{\text{m}^d}, Y_t, \pi_t} \quad \Pi_t = \pi_t \quad (2.1)$$

s.t. :

$$\pi_t = Y_t - N_t^{\text{m}^d} W_t - r_t K_t^{\text{m}^d} \quad \left(\lambda_t^{\text{FIRM}^1}\right) \quad (2.2)$$

$$Y_t = \Gamma Z_t^{(M)} K_t^{\text{m}^d \alpha} N_t^{\text{m}^d 1-\alpha} \quad \left(\lambda_t^{\text{FIRM}^2}\right) \quad (2.3)$$

2.2 First order conditions

$$-\lambda_t^{\text{FIRM}^1} r_t + \alpha \Gamma \lambda_t^{\text{FIRM}^2} Z_t^{(M)} K_t^{\text{m}^d -1+\alpha} N_t^{\text{m}^d 1-\alpha} = 0 \quad \left(K_t^{\text{m}^d}\right) \quad (2.4)$$

$$-\lambda_t^{\text{FIRM}^1} W_t + \Gamma \lambda_t^{\text{FIRM}^2} Z_t^{(M)} (1-\alpha) K_t^{\text{m}^d \alpha} N_t^{\text{m}^d -\alpha} = 0 \quad \left(N_t^{\text{m}^d}\right) \quad (2.5)$$

$$\lambda_t^{\text{FIRM}^1} - \lambda_t^{\text{FIRM}^2} = 0 \quad \left(Y_t\right) \quad (2.6)$$

$$1 - \lambda_t^{\text{FIRM}^1} = 0 \quad \left(\pi_t\right) \quad (2.7)$$

2.3 First order conditions after reduction

$$-r_t + \alpha \Gamma Z_t^{(M)} K_t^{\text{m}^d -1+\alpha} N_t^{\text{m}^d 1-\alpha} = 0 \quad \left(K_t^{\text{m}^d}\right) \quad (2.8)$$

$$-W_t + \Gamma Z_t^{(M)} (1-\alpha) K_t^{\text{m}^d \alpha} N_t^{\text{m}^d -\alpha} = 0 \quad \left(N_t^{\text{m}^d}\right) \quad (2.9)$$

3 EQUILIBRIUM

3.1 Identities

$$K_t^{\text{m}^d} = K_{t-1}^{(M)} \quad (3.1)$$

$$N_t^{\text{m}^d} = N_t^{(M)} \quad (3.2)$$

4 EXOG

4.1 Identities

$$i \in IND: \quad Z_t^{(i)} = e^{\epsilon_t^{(i)} + \psi^{(i)} \log Z_{t-1}^{(i)}} \quad (4.1)$$

5 Equilibrium relationships (before expansion and reduction)

$$-\pi_t + \Pi_t = 0 \quad (5.1)$$

$$-r_t + \alpha \Gamma Z_t^{(M)} K_t^{m^d - 1 + \alpha} N_t^{m^d 1 - \alpha} = 0 \quad (5.2)$$

$$-I_t + \sum_{i \in IND} I_t^{(i)} = 0 \quad (5.3)$$

$$-K_t + \sum_{i \in IND} K_t^{(i)} = 0 \quad (5.4)$$

$$-K_t^{m^d} + K_{t-1}^{(M)} = 0 \quad (5.5)$$

$$-N_t + \sum_{i \in IND} N_t^{(i)} = 0 \quad (5.6)$$

$$-N_t^{m^d} + N_t^{(M)} = 0 \quad (5.7)$$

$$-W_t + \Gamma Z_t^{(M)} (1 - \alpha) K_t^{m^d \alpha} N_t^{m^d - \alpha} = 0 \quad (5.8)$$

$$-Y_t + \Gamma Z_t^{(M)} K_t^{m^d \alpha} N_t^{m^d 1 - \alpha} = 0 \quad (5.9)$$

$$-C_t^{(H)} + \Gamma Z_t^{(H)} K_{t-1}^{(H) \theta} N_t^{(H) 1 - \theta} = 0 \quad (5.10)$$

$$-\pi_t + Y_t - r_t K_t^{m^d} - N_t^{m^d} W_t = 0 \quad (5.11)$$

$$U_t - \beta E_t [U_{t+1}] - \log \left(1 - \sum_{i \in IND} N_t^{(i)} \right) (1 - b) - b e^{-1} \log \left(a C_t^{(M) e} + (1 - a) C_t^{(H) e} \right) = 0 \quad (5.12)$$

$$\pi_t - C_t^{(M)} + r_t K_{t-1}^{(M)} + W_t N_t^{(M)} - \sum_{i \in IND} I_t^{(i)} = 0 \quad (5.13)$$

$$i \in IND: \quad -\lambda_t^{\text{CONSUMER}^1} + \lambda^{\text{CONSUMER}^2 (i)} = 0 \quad (5.14)$$

$$i \in IND: \quad -\lambda^{\text{CONSUMER}^2 (i)} + \beta \left(\delta^{(M, i)} E_t \left[\lambda_{t+1}^{\text{CONSUMER}^1} r_{t+1} \right] + (1 - \delta) E_t \left[\lambda^{\text{CONSUMER}^2 (i)}_{t+1} \right] + \delta^{(H, i)} \theta \Gamma K_t^{(H) - 1 + \theta} E_t \left[\lambda_{t+1}^{\text{CONSUMER}^3} Z_{t+1}^{(H)} N_{t+1}^{(H) 1 - \theta} \right] \right) = 0 \quad (5.15)$$

$$i \in IND: \quad -Z_t^{(i)} + e^{\epsilon_t^{(i)} + \psi^{(i)} \log Z_{t-1}^{(i)}} = 0 \quad (5.16)$$

$$i \in IND: \quad I_t^{(i)} - K_t^{(i)} + K_{t-1}^{(i)} (1 - \delta) = 0 \quad (5.17)$$

$$i \in IND: \quad -\delta^{(M,i)} \lambda_t^{\text{CONSUMER}^1} - \delta^{(H,i)} \lambda_t^{\text{CONSUMER}^3} + be^{-1} \left(aC_t^{(M)e} + (1-a)C_t^{(H)e} \right)^{-1} \left(\delta^{(M,i)} aeC_t^{(M)^{-1+e}} + \delta^{(H,i)} e(1-a)C_t^{(H)^{-1+e}} \right) = 0 \quad (5.18)$$

$$i \in IND: \quad -(1-b) \left(1 - \sum_{i \in IND} N_t^{(i)} \right)^{-1} + \delta^{(M,i)} \lambda_t^{\text{CONSUMER}^1} W_t + \delta^{(H,i)} \Gamma \lambda_t^{\text{CONSUMER}^3} Z_t^{(H)} (1-\theta) K_{t-1}^{(H)\theta} N_t^{(H)^{-\theta}} = 0 \quad (5.19)$$

6 Equilibrium relationships (after expansion and reduction)

$$-r_t + \alpha \Gamma Z_t^{(M)} K_{t-1}^{(M)^{-1+\alpha}} N_t^{(M)^{1-\alpha}} = 0 \quad (6.1)$$

$$-W_t + \Gamma Z_t^{(M)} (1-\alpha) K_{t-1}^{(M)\alpha} N_t^{(M)^{-\alpha}} = 0 \quad (6.2)$$

$$-Y_t + \Gamma Z_t^{(M)} K_{t-1}^{(M)\alpha} N_t^{(M)^{1-\alpha}} = 0 \quad (6.3)$$

$$-C_t^{(H)} + \Gamma Z_t^{(H)} K_{t-1}^{(H)\theta} N_t^{(H)^{1-\theta}} = 0 \quad (6.4)$$

$$-Z_t^{(H)} + e^{\epsilon_t^{(H)} + \psi^{(H)} \log Z_{t-1}^{(H)}} = 0 \quad (6.5)$$

$$-Z_t^{(M)} + e^{\epsilon_t^{(M)} + \psi^{(M)} \log Z_{t-1}^{(M)}} = 0 \quad (6.6)$$

$$\beta \left(ab E_t \left[r_{t+1} \left(aC_{t+1}^{(M)e} + (1-a)C_{t+1}^{(H)e} \right)^{-1} C_{t+1}^{(M)^{-1+e}} \right] + ab(1-\delta) E_t \left[\left(aC_{t+1}^{(M)e} + (1-a)C_{t+1}^{(H)e} \right)^{-1} C_{t+1}^{(M)^{-1+e}} \right] \right) - ab \left(aC_t^{(M)e} + (1-a)C_t^{(H)e} \right)^{-1} C_t^{(M)^{-1+e}} = 0 \quad (6.7)$$

$$\beta \left(ab(1-\delta) E_t \left[\left(aC_{t+1}^{(M)e} + (1-a)C_{t+1}^{(H)e} \right)^{-1} C_{t+1}^{(M)^{-1+e}} \right] + b\theta \Gamma (1-a) K_t^{(H)^{-1+\theta}} E_t \left[Z_{t+1}^{(H)} \left(aC_{t+1}^{(M)e} + (1-a)C_{t+1}^{(H)e} \right)^{-1} C_{t+1}^{(H)^{-1+e}} N_{t+1}^{(H)^{1-\theta}} \right] \right) - ab \left(aC_t^{(M)e} + (1-a)C_t^{(H)e} \right)^{-1} C_t^{(M)^{-1+e}} = 0 \quad (6.8)$$

$$-(1-b) \left(1 - N_t^{(H)} - N_t^{(M)} \right)^{-1} + abW_t \left(aC_t^{(M)e} + (1-a)C_t^{(H)e} \right)^{-1} C_t^{(M)^{-1+e}} = 0 \quad (6.9)$$

$$-(1-b) \left(1 - N_t^{(H)} - N_t^{(M)} \right)^{-1} + b\Gamma Z_t^{(H)} (1-a)(1-\theta) \left(aC_t^{(M)e} + (1-a)C_t^{(H)e} \right)^{-1} K_{t-1}^{(H)\theta} C_t^{(H)^{-1+e}} N_t^{(H)^{-\theta}} = 0 \quad (6.10)$$

$$-I_t + I_t^{(H)} + I_t^{(M)} = 0 \quad (6.11)$$

$$-K_t + K_t^{(H)} + K_t^{(M)} = 0 \quad (6.12)$$

$$-N_t + N_t^{(H)} + N_t^{(M)} = 0 \quad (6.13)$$

$$I_t^{(H)} - K_t^{(H)} + K_{t-1}^{(H)} (1 - \delta) = 0 \quad (6.14)$$

$$I_t^{(M)} - K_t^{(M)} + K_{t-1}^{(M)} (1 - \delta) = 0 \quad (6.15)$$

$$U_t - \beta E_t [U_{t+1}] - \log \left(1 - N_t^{(H)} - N_t^{(M)} \right) (1 - b) - be^{-1} \log \left(aC_t^{(M)e} + (1 - a) C_t^{(H)e} \right) = 0 \quad (6.16)$$

$$Y_t - C_t^{(M)} - I_t^{(H)} - I_t^{(M)} = 0 \quad (6.17)$$

7 Steady state relationships (before expansion and reduction)

$$-\pi_{ss} + \Pi_{ss} = 0 \quad (7.1)$$

$$-r_{ss} + \alpha \Gamma Z_{ss}^{(M)} K_{ss}^{m^d - 1 + \alpha} N_{ss}^{m^d 1 - \alpha} = 0 \quad (7.2)$$

$$-I_{ss} + \sum_{i \in IND} I_{ss}^{(i)} = 0 \quad (7.3)$$

$$-K_{ss} + \sum_{i \in IND} K_{ss}^{(i)} = 0 \quad (7.4)$$

$$-K_{ss}^{m^d} + K_{ss}^{(M)} = 0 \quad (7.5)$$

$$-N_{ss} + \sum_{i \in IND} N_{ss}^{(i)} = 0 \quad (7.6)$$

$$-N_{ss}^{m^d} + N_{ss}^{(M)} = 0 \quad (7.7)$$

$$-W_{ss} + \Gamma Z_{ss}^{(M)} (1 - \alpha) K_{ss}^{m^d \alpha} N_{ss}^{m^d - \alpha} = 0 \quad (7.8)$$

$$-Y_{ss} + \Gamma Z_{ss}^{(M)} K_{ss}^{m^d \alpha} N_{ss}^{m^d 1 - \alpha} = 0 \quad (7.9)$$

$$-C_{ss}^{(H)} + \Gamma Z_{ss}^{(H)} K_{ss}^{(H)\theta} N_{ss}^{(H)1 - \theta} = 0 \quad (7.10)$$

$$-\pi_{ss} + Y_{ss} - r_{ss} K_{ss}^{m^d} - N_{ss}^{m^d} W_{ss} = 0 \quad (7.11)$$

$$U_{ss} - \beta U_{ss} - \log \left(1 - \sum_{i \in IND} N_{ss}^{(i)} \right) (1 - b) - be^{-1} \log \left(aC_{ss}^{(M)e} + (1 - a) C_{ss}^{(H)e} \right) = 0 \quad (7.12)$$

$$\pi_{ss} - C_{ss}^{(M)} + r_{ss} K_{ss}^{(M)} + W_{ss} N_{ss}^{(M)} - \sum_{i \in IND} I_{ss}^{(i)} = 0 \quad (7.13)$$

$$i \in IND: \quad -\lambda_{ss}^{CONSUMER^1} + \lambda_{ss}^{CONSUMER^2(i)} = 0 \quad (7.14)$$

$$i \in IND: \quad -\lambda_{ss}^{CONSUMER^2(i)} + \beta \left(\lambda_{ss}^{CONSUMER^2(i)} (1 - \delta) + \delta^{(M,i)} \lambda_{ss}^{CONSUMER^1} r_{ss} + \delta^{(H,i)} \theta \Gamma \lambda_{ss}^{CONSUMER^3} Z_{ss}^{(H)} K_{ss}^{(H)-1+\theta} N_{ss}^{(H)1-\theta} \right) = 0 \quad (7.15)$$

$$i \in IND: \quad -Z_{ss}^{(i)} + e^{\epsilon_{ss}^{(i)} + \psi^{(i)} \log Z_{ss}^{(i)}} = 0 \quad (7.16)$$

$$i \in IND: \quad I_{ss}^{(i)} - K_{ss}^{(i)} + K_{ss}^{(i)} (1 - \delta) = 0 \quad (7.17)$$

$$i \in IND: \quad -\delta^{(M,i)} \lambda_{ss}^{\text{CONSUMER}^1} - \delta^{(H,i)} \lambda_{ss}^{\text{CONSUMER}^3} + be^{-1} \left(aC_{ss}^{(M)e} + (1-a)C_{ss}^{(H)e} \right)^{-1} \left(\delta^{(M,i)} aeC_{ss}^{(M)-1+e} + \delta^{(H,i)} e(1-a)C_{ss}^{(H)-1+e} \right) = 0 \quad (7.18)$$

$$i \in IND: \quad -(1-b) \left(1 - \sum_{i \in IND} N_{ss}^{(i)} \right)^{-1} + \delta^{(M,i)} \lambda_{ss}^{\text{CONSUMER}^1} W_{ss} + \delta^{(H,i)} \Gamma \lambda_{ss}^{\text{CONSUMER}^3} Z_{ss}^{(H)} (1-\theta) K_{ss}^{(H)\theta} N_{ss}^{(H)-\theta} = 0 \quad (7.19)$$

8 Steady state relationships (after expansion and reduction)

$$-r_{ss} + \alpha \Gamma Z_{ss}^{(M)} K_{ss}^{(M)-1+\alpha} N_{ss}^{(M)1-\alpha} = 0 \quad (8.1)$$

$$-W_{ss} + \Gamma Z_{ss}^{(M)} (1-\alpha) K_{ss}^{(M)\alpha} N_{ss}^{(M)-\alpha} = 0 \quad (8.2)$$

$$-Y_{ss} + \Gamma Z_{ss}^{(M)} K_{ss}^{(M)\alpha} N_{ss}^{(M)1-\alpha} = 0 \quad (8.3)$$

$$-C_{ss}^{(H)} + \Gamma Z_{ss}^{(H)} K_{ss}^{(H)\theta} N_{ss}^{(H)1-\theta} = 0 \quad (8.4)$$

$$-Z_{ss}^{(H)} + e^{\psi^{(H)} \log Z_{ss}^{(H)}} = 0 \quad (8.5)$$

$$-Z_{ss}^{(M)} + e^{\psi^{(M)} \log Z_{ss}^{(M)}} = 0 \quad (8.6)$$

$$\beta \left(abr_{ss} \left(aC_{ss}^{(M)e} + (1-a)C_{ss}^{(H)e} \right)^{-1} C_{ss}^{(M)-1+e} + ab(1-\delta) \left(aC_{ss}^{(M)e} + (1-a)C_{ss}^{(H)e} \right)^{-1} C_{ss}^{(M)-1+e} \right) - ab \left(aC_{ss}^{(M)e} + (1-a)C_{ss}^{(H)e} \right)^{-1} C_{ss}^{(M)-1+e} = 0 \quad (8.7)$$

$$\beta \left(ab(1-\delta) \left(aC_{ss}^{(M)e} + (1-a)C_{ss}^{(H)e} \right)^{-1} C_{ss}^{(M)-1+e} + b\theta \Gamma Z_{ss}^{(H)} (1-a) \left(aC_{ss}^{(M)e} + (1-a)C_{ss}^{(H)e} \right)^{-1} C_{ss}^{(H)-1+e} K_{ss}^{(H)-1+\theta} N_{ss}^{(H)1-\theta} \right) - ab \left(aC_{ss}^{(M)e} + (1-a)C_{ss}^{(H)e} \right)^{-1} C_{ss}^{(M)-1+e} = 0 \quad (8.8)$$

$$-(1-b) \left(1 - N_{ss}^{(H)} - N_{ss}^{(M)} \right)^{-1} + abW_{ss} \left(aC_{ss}^{(M)e} + (1-a)C_{ss}^{(H)e} \right)^{-1} C_{ss}^{(M)-1+e} = 0 \quad (8.9)$$

$$-(1-b) \left(1 - N_{ss}^{(H)} - N_{ss}^{(M)} \right)^{-1} + b\Gamma Z_{ss}^{(H)} (1-a)(1-\theta) \left(aC_{ss}^{(M)e} + (1-a)C_{ss}^{(H)e} \right)^{-1} C_{ss}^{(H)-1+e} K_{ss}^{(H)\theta} N_{ss}^{(H)-\theta} = 0 \quad (8.10)$$

$$-I_{ss} + I_{ss}^{(H)} + I_{ss}^{(M)} = 0 \quad (8.11)$$

$$-K_{ss} + K_{ss}^{(H)} + K_{ss}^{(M)} = 0 \quad (8.12)$$

$$-N_{ss} + N_{ss}^{(H)} + N_{ss}^{(M)} = 0 \quad (8.13)$$

$$I_{ss}^{(H)} - K_{ss}^{(H)} + K_{ss}^{(H)} (1-\delta) = 0 \quad (8.14)$$

$$I_{ss}^{(M)} - K_{ss}^{(M)} + K_{ss}^{(M)} (1-\delta) = 0 \quad (8.15)$$

$$U_{ss} - \beta U_{ss} - \log \left(1 - N_{ss}^{(H)} - N_{ss}^{(M)} \right) (1-b) - be^{-1} \log \left(aC_{ss}^{(M)e} + (1-a)C_{ss}^{(H)e} \right) = 0 \quad (8.16)$$

$$Y_{ss} - C_{ss}^{(M)} - I_{ss}^{(H)} - I_{ss}^{(M)} = 0 \quad (8.17)$$

9 Parameter settings

$$a = 0.337 \tag{9.1}$$

$$\alpha = 0.36 \tag{9.2}$$

$$b = 0.63 \tag{9.3}$$

$$\beta = 0.99 \tag{9.4}$$

$$\delta = 0.025 \tag{9.5}$$

$$e = 0.8 \tag{9.6}$$

$$\theta = 0.08 \tag{9.7}$$

$$\Gamma = 1 \tag{9.8}$$

$$\psi^{(H)} = 0.95 \tag{9.9}$$

$$\psi^{(M)} = 0.95 \tag{9.10}$$

10 Steady-state values

	Steady-state value
r	0.0351
I	0.3143
K	12.5726
N	0.6102
U	-79.6929
W	2.3706
Y	1.0367
$C^{(H)}$	0.3805
$C^{(M)}$	0.7224
$I^{(H)}$	0.0485
$I^{(M)}$	0.2658
$K^{(H)}$	1.9397
$K^{(M)}$	10.6329
$N^{(H)}$	0.3303
$N^{(M)}$	0.2799
$Z^{(H)}$	1
$Z^{(M)}$	1

11 The solution of the 1st order perturbation

Matrix P

$$\begin{matrix} K_t^{(H)} \\ K_t^{(M)} \\ Z_t^{(H)} \\ Z_t^{(M)} \end{matrix} \begin{pmatrix} K_{t-1}^{(H)} & K_{t-1}^{(M)} & Z_{t-1}^{(H)} & Z_{t-1}^{(M)} \\ 0.0826 & 0.4683 & 2.0323 & -2.6403 \\ 0.1545 & 0.8762 & -0.3729 & 0.6255 \\ 0 & 0 & 0.95 & 0 \\ 0 & 0 & 0 & 0.95 \end{pmatrix}$$

Matrix Q

$$\begin{matrix} K^{(H)} \\ K^{(M)} \\ Z^{(H)} \\ Z^{(M)} \end{matrix} \begin{pmatrix} \epsilon^{(H)} & \epsilon^{(M)} \\ 2.1393 & -2.7792 \\ -0.3926 & 0.6584 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Matrix R

$$\begin{matrix} r_t \\ I_t \\ K_t \\ N_t \\ U_t \\ W_t \\ Y_t \\ C_t^{(H)} \\ C_t^{(M)} \\ I_t^{(H)} \\ I_t^{(M)} \\ N_t^{(H)} \\ N_t^{(M)} \end{matrix} \begin{pmatrix} K_{t-1}^{(H)} & K_{t-1}^{(M)} & Z_{t-1}^{(H)} & Z_{t-1}^{(M)} \\ -0.08 & -0.4894 & -0.6218 & 1.96 \\ -0.2798 & -0.4533 & -0.0746 & 4.867 \\ 0.1434 & 0.8132 & -0.0019 & 0.1217 \\ -0.0155 & -0.0751 & 0.0429 & 0.226 \\ 0.0098 & 0.054 & 0.0683 & 0.0832 \\ 0.045 & 0.2753 & 0.3497 & 0.3819 \\ -0.08 & 0.5106 & -0.6218 & 1.96 \\ 0.1511 & -0.3112 & 1.7804 & -0.8463 \\ 0.0069 & 0.93 & -0.8599 & 0.6952 \\ -35.696 & 18.734 & 81.2939 & -105.6101 \\ 6.1809 & -3.9534 & -14.918 & 25.0205 \\ 0.0772 & -0.3382 & 0.9026 & -0.9199 \\ -0.125 & 0.2353 & -0.9715 & 1.5781 \end{pmatrix}$$

Matrix S

$$\begin{array}{c}
 \epsilon^{(H)} \\
 \epsilon^{(M)} \\
 r \\
 I \\
 K \\
 N \\
 U \\
 W \\
 Y \\
 C^{(H)} \\
 C^{(M)} \\
 I^{(H)} \\
 I^{(M)} \\
 N^{(H)} \\
 N^{(M)}
 \end{array}
 \begin{pmatrix}
 -0.6545 & 2.0631 \\
 -0.0785 & 5.1231 \\
 -0.002 & 0.1281 \\
 0.0452 & 0.2379 \\
 0.0719 & 0.0875 \\
 0.3682 & 0.402 \\
 -0.6545 & 2.0631 \\
 1.8741 & -0.8908 \\
 -0.9051 & 0.7318 \\
 85.5725 & -111.1686 \\
 -15.7031 & 26.3373 \\
 0.9501 & -0.9683 \\
 -1.0227 & 1.6612
 \end{pmatrix}$$

12 Model statistics

12.1 Basic statistics

	Steady-state value	Std. dev.	Variance	Loglin
$C^{(M)}$	0.7224	0.9767	0.954	Y
$C^{(H)}$	0.3805	1.52	2.3104	Y
Y	1.0367	1.7868	3.1926	Y
$I^{(M)}$	0.2658	12.888	166.0993	Y
$I^{(H)}$	0.0485	57.2629	3279.0405	Y
$K^{(M)}$	10.6329	0.6337	0.4016	Y
$K^{(H)}$	1.9397	1.9051	3.6295	Y
$N^{(M)}$	0.2799	1.3761	1.8936	Y
$N^{(H)}$	0.3303	0.9286	0.8623	Y
W	2.3706	0.6601	0.4358	Y

12.2 Correlation matrix

	W	Y	$C^{(H)}$	$C^{(M)}$	$I^{(H)}$	$I^{(M)}$	$K^{(H)}$	$K^{(M)}$	$N^{(H)}$	$N^{(M)}$
W	1	0.735	0.421	0.056	-0.028	0.409	-0.271	0.511	-0.148	0.475
Y		1	-0.288	0.622	-0.16	0.542	-0.829	0.779	-0.768	0.946
$C^{(H)}$			1	-0.843	0.084	-0.05	0.657	-0.438	0.833	-0.575
$C^{(M)}$				1	0.021	0.114	-0.704	0.842	-0.933	0.781
$I^{(H)}$					1	-0.901	0.409	-0.068	0.135	-0.194
$I^{(M)}$						1	-0.634	0.303	-0.332	0.508
$K^{(H)}$							1	-0.592	0.906	-0.946
$K^{(M)}$								1	-0.76	0.766
$N^{(H)}$									1	-0.927
$N^{(M)}$										1

12.3 Cross correlations with the reference variable (Y)

	$\sigma[\cdot]$ rel. to $\sigma[Y]$	Y_{t-5}	Y_{t-4}	Y_{t-3}	Y_{t-2}	Y_{t-1}	Y_t	Y_{t+1}	Y_{t+2}	Y_{t+3}	Y_{t+4}	Y_{t+5}
W_t	0.369	-0.032	0.068	0.197	0.356	0.547	0.735	0.568	0.415	0.281	0.166	0.071
Y_t	1	0.008	0.146	0.32	0.533	0.787	1	0.787	0.533	0.32	0.146	0.008
$C_t^{(H)}$	0.851	0.044	-0.001	-0.06	-0.136	-0.229	-0.288	-0.318	-0.238	-0.168	-0.108	-0.057
$C_t^{(M)}$	0.547	-0.194	-0.098	0.033	0.204	0.42	0.622	0.68	0.584	0.483	0.383	0.287
$I_t^{(H)}$	32.048	-0.124	-0.156	-0.192	-0.23	-0.27	-0.16	0.372	0.307	0.246	0.192	0.144
$I_t^{(M)}$	7.213	0.145	0.226	0.322	0.433	0.558	0.542	-0.016	-0.08	-0.126	-0.155	-0.172
$K_t^{(H)}$	1.066	-0.12	-0.235	-0.373	-0.537	-0.727	-0.829	-0.528	-0.284	-0.092	0.054	0.161
$K_t^{(M)}$	0.355	-0.27	-0.149	0.019	0.238	0.516	0.779	0.751	0.692	0.61	0.516	0.416
$N_t^{(H)}$	0.52	0.04	-0.07	-0.212	-0.387	-0.6	-0.768	-0.677	-0.484	-0.318	-0.179	-0.065
$N_t^{(M)}$	0.77	0.026	0.157	0.322	0.522	0.76	0.946	0.75	0.493	0.281	0.11	-0.024

12.4 Autocorrelations

	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5
W	0.74	0.511	0.317	0.156	0.026
Y	0.787	0.533	0.32	0.146	0.008
$C^{(H)}$	0.771	0.512	0.298	0.124	-0.011
$C^{(M)}$	0.852	0.622	0.418	0.241	0.092
$I^{(H)}$	-0.078	-0.074	-0.068	-0.061	-0.054
$I^{(M)}$	0.076	0.032	-0.003	-0.029	-0.048
$K^{(H)}$	0.711	0.467	0.266	0.105	-0.021
$K^{(M)}$	0.868	0.716	0.556	0.398	0.247
$N^{(H)}$	0.82	0.555	0.333	0.152	0.008
$N^{(M)}$	0.803	0.537	0.316	0.136	-0.005

12.5 Variance decomposition

	$\epsilon^{(H)}$	$\epsilon^{(M)}$
W	0.796	0.204
Y	0.136	0.864
$C^{(H)}$	0.616	0.384
$C^{(M)}$	0.238	0.762
$I^{(H)}$	0.015	0.985
$I^{(M)}$	0.053	0.947
$K^{(H)}$	0.023	0.977
$K^{(M)}$	0.198	0.802
$N^{(H)}$	0.106	0.894
$N^{(M)}$	0.003	0.997

13 Impulse response functions

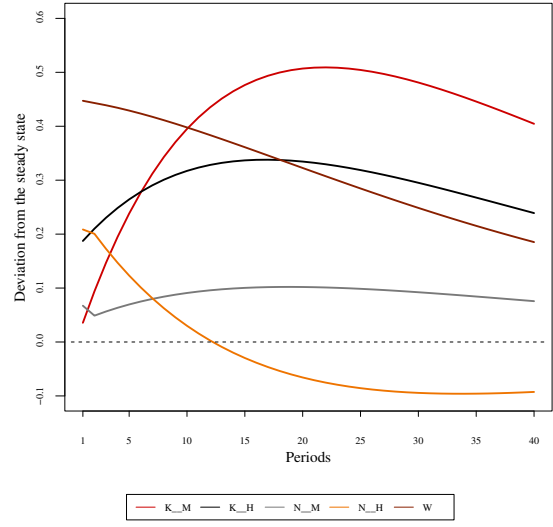
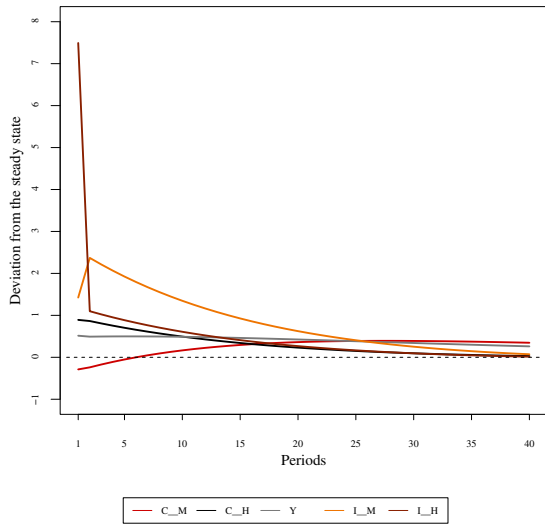


Figure 1: Impulse responses $(C^{(M)}, C^{(H)}, Y, I^{(M)}, I^{(H)})$ to $\epsilon^{(H)}$ shock

Figure 2: Impulse responses $(K^{(M)}, K^{(H)}, N^{(M)}, N^{(H)}, W)$ to $\epsilon^{(H)}$ shock

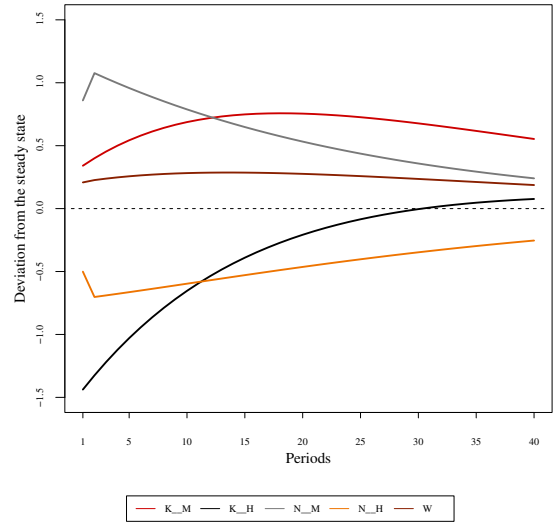
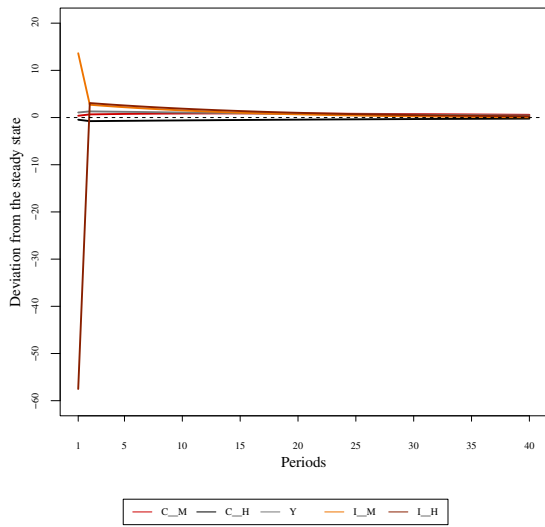


Figure 3: Impulse responses $(C^{(M)}, C^{(H)}, Y, I^{(M)}, I^{(H)})$ to $\epsilon^{(M)}$ shock

Figure 4: Impulse responses $(K^{(M)}, K^{(H)}, N^{(M)}, N^{(H)}, W)$ to $\epsilon^{(M)}$ shock