

## 1 CONSUMER

### 1.1 Optimisation problem

$$\max_{K_t^m, K_t^h, C_t^m, C_t^h, N_t^m, N_t^h, I_t^m, I_t^h} U_t = \beta E_t [U_{t+1}] + \log(1 - N_t^m - N_t^h) (1 - b) + be^{-1} \log(aC_t^{me} + (1 - a)C_t^{he}) \quad (1.1)$$

s.t. :

$$C_t^m + I_t^m + I_t^h = \pi_t + K_{t-1}^m r_t + N_t^m W_t \quad (\lambda_t^{\text{CONSUMER}^1}) \quad (1.2)$$

$$K_t^m = I_t^m + K_{t-1}^m (1 - \delta) \quad (\lambda_t^{\text{CONSUMER}^2}) \quad (1.3)$$

$$K_t^h = I_t^h + K_{t-1}^h (1 - \delta) \quad (\lambda_t^{\text{CONSUMER}^3}) \quad (1.4)$$

$$C_t^h = \Gamma Z_t^h K_{t-1}^h{}^\theta N_t^h{}^{1-\theta} \quad (\lambda_t^{\text{CONSUMER}^4}) \quad (1.5)$$

### 1.2 Identities

$$K_t = K_t^m + K_t^h \quad (1.6)$$

$$I_t = I_t^m + I_t^h \quad (1.7)$$

$$N_t = N_t^m + N_t^h \quad (1.8)$$

### 1.3 First order conditions

$$-\lambda_t^{\text{CONSUMER}^2} + \beta \left( (1 - \delta) E_t [\lambda_{t+1}^{\text{CONSUMER}^2}] + E_t [\lambda_{t+1}^{\text{CONSUMER}^1} r_{t+1}] \right) = 0 \quad (K_t^m) \quad (1.9)$$

$$-\lambda_t^{\text{CONSUMER}^3} + \beta \left( (1 - \delta) E_t [\lambda_{t+1}^{\text{CONSUMER}^3}] + \theta \Gamma K_t^h{}^{-1+\theta} E_t [\lambda_{t+1}^{\text{CONSUMER}^4} Z_{t+1}^h N_{t+1}^h{}^{1-\theta}] \right) = 0 \quad (K_t^h) \quad (1.10)$$

$$-\lambda_t^{\text{CONSUMER}^1} + ab \left( aC_t^{me} + (1 - a)C_t^{he} \right)^{-1} C_t^{m-1+e} = 0 \quad (C_t^m) \quad (1.11)$$

$$-\lambda_t^{\text{CONSUMER}^4} + b(1 - a) \left( aC_t^{me} + (1 - a)C_t^{he} \right)^{-1} C_t^{h-1+e} = 0 \quad (C_t^h) \quad (1.12)$$

$$\lambda_t^{\text{CONSUMER}^1} W_t - (1-b)(1-N_t^m - N_t^h)^{-1} = 0 \quad (N_t^m) \quad (1.13)$$

$$-(1-b)(1-N_t^m - N_t^h)^{-1} + \Gamma \lambda_t^{\text{CONSUMER}^4} Z_t^h (1-\theta) K_{t-1}^h{}^\theta N_t^h{}^{-\theta} = 0 \quad (N_t^h) \quad (1.14)$$

$$-\lambda_t^{\text{CONSUMER}^1} + \lambda_t^{\text{CONSUMER}^2} = 0 \quad (I_t^m) \quad (1.15)$$

$$-\lambda_t^{\text{CONSUMER}^1} + \lambda_t^{\text{CONSUMER}^3} = 0 \quad (I_t^h) \quad (1.16)$$

## 2 FIRM

### 2.1 Optimisation problem

$$\max_{K_t^{\text{m}^d}, N_t^{\text{m}^d}, Y_t, \pi_t} \Pi_t = \pi_t \quad (2.1)$$

s.t. :

$$\pi_t = Y_t - N_t^{\text{m}^d} W_t - r_t K_t^{\text{m}^d} \quad (\lambda_t^{\text{FIRM}^1}) \quad (2.2)$$

$$Y_t = \Gamma Z_t^m K_t^{\text{m}^d \alpha} N_t^{\text{m}^d 1-\alpha} \quad (\lambda_t^{\text{FIRM}^2}) \quad (2.3)$$

### 2.2 First order conditions

$$-\lambda_t^{\text{FIRM}^1} r_t + \alpha \Gamma \lambda_t^{\text{FIRM}^2} Z_t^m K_t^{\text{m}^d -1+\alpha} N_t^{\text{m}^d 1-\alpha} = 0 \quad (K_t^{\text{m}^d}) \quad (2.4)$$

$$-\lambda_t^{\text{FIRM}^1} W_t + \Gamma \lambda_t^{\text{FIRM}^2} Z_t^m (1-\alpha) K_t^{\text{m}^d \alpha} N_t^{\text{m}^d -\alpha} = 0 \quad (N_t^{\text{m}^d}) \quad (2.5)$$

$$\lambda_t^{\text{FIRM}^1} - \lambda_t^{\text{FIRM}^2} = 0 \quad (Y_t) \quad (2.6)$$

$$1 - \lambda_t^{\text{FIRM}^1} = 0 \quad (\pi_t) \quad (2.7)$$

### 2.3 First order conditions after reduction

$$-r_t + \alpha \Gamma Z_t^m K_t^{\text{m}^d -1+\alpha} N_t^{\text{m}^d 1-\alpha} = 0 \quad (K_t^{\text{m}^d}) \quad (2.8)$$

$$-W_t + \Gamma Z_t^m (1-\alpha) K_t^{\text{m}^d \alpha} N_t^{\text{m}^d -\alpha} = 0 \quad (N_t^{\text{m}^d}) \quad (2.9)$$

### 3 EQUILIBRIUM

#### 3.1 Identities

$$K_t^{\text{m}^d} = K_{t-1}^{\text{m}} \quad (3.1)$$

$$N_t^{\text{m}^d} = N_t^{\text{m}} \quad (3.2)$$

### 4 EXOG

#### 4.1 Identities

$$Z_t^{\text{h}} = e^{\epsilon_t^{\text{h}} + \psi \log Z_{t-1}^{\text{h}}} \quad (4.1)$$

$$Z_t^{\text{m}} = e^{\epsilon_t^{\text{m}} + \phi \log Z_{t-1}^{\text{m}}} \quad (4.2)$$

### 5 Equilibrium relationships (after reduction)

$$-r_t + \alpha \Gamma Z_t^{\text{m}} K_{t-1}^{\text{m}^{-1+\alpha}} N_t^{\text{m}^{1-\alpha}} = 0 \quad (5.1)$$

$$-C_t^{\text{h}} + \Gamma Z_t^{\text{h}} K_{t-1}^{\text{h}^{\theta}} N_t^{\text{h}^{1-\theta}} = 0 \quad (5.2)$$

$$-W_t + \Gamma Z_t^{\text{m}} (1 - \alpha) K_{t-1}^{\text{m}^{\alpha}} N_t^{\text{m}^{-\alpha}} = 0 \quad (5.3)$$

$$-Y_t + \Gamma Z_t^{\text{m}} K_{t-1}^{\text{m}^{\alpha}} N_t^{\text{m}^{1-\alpha}} = 0 \quad (5.4)$$

$$-Z_t^{\text{h}} + e^{\epsilon_t^{\text{h}} + \psi \log Z_{t-1}^{\text{h}}} = 0 \quad (5.5)$$

$$-Z_t^{\text{m}} + e^{\epsilon_t^{\text{m}} + \phi \log Z_{t-1}^{\text{m}}} = 0 \quad (5.6)$$

$$\beta \left( ab \mathbb{E}_t \left[ r_{t+1} \left( a C_{t+1}^{\text{m}^e} + (1-a) C_{t+1}^{\text{h}^e} \right)^{-1} C_{t+1}^{\text{m}^{-1+e}} \right] + ab(1-\delta) \mathbb{E}_t \left[ \left( a C_{t+1}^{\text{m}^e} + (1-a) C_{t+1}^{\text{h}^e} \right)^{-1} C_{t+1}^{\text{m}^{-1+e}} \right] \right) - ab \left( a C_t^{\text{m}^e} + (1-a) C_t^{\text{h}^e} \right)^{-1} C_t^{\text{m}^{-1+e}} = 0 \quad (5.7)$$

$$\beta \left( ab(1-\delta) \mathbb{E}_t \left[ \left( a C_{t+1}^{\text{m}^e} + (1-a) C_{t+1}^{\text{h}^e} \right)^{-1} C_{t+1}^{\text{m}^{-1+e}} \right] + b\theta \Gamma (1-a) K_t^{\text{h}^{-1+\theta}} \mathbb{E}_t \left[ Z_{t+1}^{\text{h}} \left( a C_{t+1}^{\text{m}^e} + (1-a) C_{t+1}^{\text{h}^e} \right)^{-1} C_{t+1}^{\text{h}^{-1+e}} N_{t+1}^{\text{h}^{1-\theta}} \right] \right) - ab \left( a C_t^{\text{m}^e} + (1-a) C_t^{\text{h}^e} \right)^{-1} C_t^{\text{m}^{-1+e}} = 0 \quad (5.8)$$

$$-(1-b)(1-N_t^m - N_t^h)^{-1} + abW_t \left( aC_t^{me} + (1-a)C_t^{he} \right)^{-1} C_t^{m-1+e} = 0 \quad (5.9)$$

$$-(1-b)(1-N_t^m - N_t^h)^{-1} + b\Gamma Z_t^h (1-a)(1-\theta) \left( aC_t^{me} + (1-a)C_t^{he} \right)^{-1} K_{t-1}^h{}^\theta C_t^{h-1+e} N_t^{h-\theta} = 0 \quad (5.10)$$

$$-I_t + I_t^m + I_t^h = 0 \quad (5.11)$$

$$I_t^m - K_t^m + K_{t-1}^m (1-\delta) = 0 \quad (5.12)$$

$$I_t^h - K_t^h + K_{t-1}^h (1-\delta) = 0 \quad (5.13)$$

$$-K_t + K_t^m + K_t^h = 0 \quad (5.14)$$

$$-N_t + N_t^m + N_t^h = 0 \quad (5.15)$$

$$-C_t^m - I_t^m - I_t^h + Y_t = 0 \quad (5.16)$$

$$U_t - \beta E_t [U_{t+1}] - \log(1 - N_t^m - N_t^h) (1-b) - be^{-1} \log \left( aC_t^{me} + (1-a)C_t^{he} \right) = 0 \quad (5.17)$$

## 6 Steady state relationships (after reduction)

$$-r_{ss} + \alpha \Gamma Z_{ss}^m K_{ss}^{m-1+\alpha} N_{ss}^{m1-\alpha} = 0 \quad (6.1)$$

$$-C_{ss}^h + \Gamma Z_{ss}^h K_{ss}^{h\theta} N_{ss}^{h1-\theta} = 0 \quad (6.2)$$

$$-W_{ss} + \Gamma Z_{ss}^m (1-\alpha) K_{ss}^{m\alpha} N_{ss}^{m-\alpha} = 0 \quad (6.3)$$

$$-Y_{ss} + \Gamma Z_{ss}^m K_{ss}^{m\alpha} N_{ss}^{m1-\alpha} = 0 \quad (6.4)$$

$$-Z_{ss}^h + e^{\psi \log Z_{ss}^h} = 0 \quad (6.5)$$

$$-Z_{ss}^m + e^{\phi \log Z_{ss}^m} = 0 \quad (6.6)$$

$$\beta \left( abr_{ss} \left( aC_{ss}^{me} + (1-a)C_{ss}^{he} \right)^{-1} C_{ss}^{m-1+e} + ab(1-\delta) \left( aC_{ss}^{me} + (1-a)C_{ss}^{he} \right)^{-1} C_{ss}^{m-1+e} \right) - ab \left( aC_{ss}^{me} + (1-a)C_{ss}^{he} \right)^{-1} C_{ss}^{m-1+e} = 0 \quad (6.7)$$

$$\beta \left( ab(1-\delta) \left( aC_{ss}^{me} + (1-a)C_{ss}^{he} \right)^{-1} C_{ss}^{m-1+e} + b\theta\Gamma Z_{ss}^h (1-a) \left( aC_{ss}^{me} + (1-a)C_{ss}^{he} \right)^{-1} C_{ss}^{h-1+e} K_{ss}^{h-1+\theta} N_{ss}^{h-1-\theta} \right) - ab \left( aC_{ss}^{me} + (1-a)C_{ss}^{he} \right)^{-1} C_{ss}^{m-1+e} = 0 \quad (6.8)$$

$$-(1-b) \left( 1 - N_{ss}^m - N_{ss}^h \right)^{-1} + abW_{ss} \left( aC_{ss}^{me} + (1-a)C_{ss}^{he} \right)^{-1} C_{ss}^{m-1+e} = 0 \quad (6.9)$$

$$-(1-b) \left( 1 - N_{ss}^m - N_{ss}^h \right)^{-1} + b\Gamma Z_{ss}^h (1-a)(1-\theta) \left( aC_{ss}^{me} + (1-a)C_{ss}^{he} \right)^{-1} C_{ss}^{h-1+e} K_{ss}^{h\theta} N_{ss}^{h-\theta} = 0 \quad (6.10)$$

$$-I_{ss} + I_{ss}^m + I_{ss}^h = 0 \quad (6.11)$$

$$I_{ss}^m - K_{ss}^m + K_{ss}^m (1-\delta) = 0 \quad (6.12)$$

$$I_{ss}^h - K_{ss}^h + K_{ss}^h (1-\delta) = 0 \quad (6.13)$$

$$-K_{ss} + K_{ss}^m + K_{ss}^h = 0 \quad (6.14)$$

$$-N_{ss} + N_{ss}^m + N_{ss}^h = 0 \quad (6.15)$$

$$-C_{ss}^m - I_{ss}^m - I_{ss}^h + Y_{ss} = 0 \quad (6.16)$$

$$U_{ss} - \beta U_{ss} - \log(1 - N_{ss}^m - N_{ss}^h)(1-b) - be^{-1} \log \left( aC_{ss}^{me} + (1-a)C_{ss}^{he} \right) = 0 \quad (6.17)$$

## 7 Parameter settings

$$a = 0.337 \quad (7.1)$$

$$\alpha = 0.36 \quad (7.2)$$

$$b = 0.63 \quad (7.3)$$

$$\beta = 0.99 \tag{7.4}$$

$$\delta = 0.025 \tag{7.5}$$

$$e = 0.8 \tag{7.6}$$

$$\phi = 0.95 \tag{7.7}$$

$$\psi = 0.95 \tag{7.8}$$

$$\theta = 0.08 \tag{7.9}$$

$$\Gamma = 1 \tag{7.10}$$

## 8 Steady-state values

	Steady-state value
$r$	0.0351
$C^m$	0.7224
$C^h$	0.3805
$I$	0.3143
$I^m$	0.2658
$I^h$	0.0485
$K$	12.5726
$K^m$	10.6329
$K^h$	1.9397
$N$	0.6102
$N^m$	0.2799
$N^h$	0.3303
$U$	-79.6929
$W$	2.3706
$Y$	1.0367
$Z^h$	1
$Z^m$	1

## 9 The solution of the 1st order perturbation

Matrix  $P$

$$\begin{matrix} K_t^m \\ K_t^h \\ Z_t^h \\ Z_t^m \end{matrix} \begin{pmatrix} K_{t-1}^m & K_{t-1}^h & Z_{t-1}^h & Z_{t-1}^m \\ 0.8762 & 0.1545 & -0.3729 & 0.6255 \\ 0.4683 & 0.0826 & 2.0323 & -2.6403 \\ 0 & 0 & 0.95 & 0 \\ 0 & 0 & 0 & 0.95 \end{pmatrix}$$

Matrix  $Q$

$$\begin{matrix} K^m \\ K^h \\ Z^h \\ Z^m \end{matrix} \begin{pmatrix} \epsilon^h & \epsilon^m \\ -0.3926 & 0.6584 \\ 2.1393 & -2.7792 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Matrix  $R$

$$\begin{matrix} r_t \\ C_t^m \\ C_t^h \\ I_t \\ I_t^m \\ I_t^h \\ K_t \\ N_t \\ N_t^m \\ N_t^h \\ U_t \\ W_t \\ Y_t \end{matrix} \begin{pmatrix} K_{t-1}^m & K_{t-1}^h & Z_{t-1}^h & Z_{t-1}^m \\ -0.4894 & -0.08 & -0.6218 & 1.96 \\ 0.93 & 0.0069 & -0.8599 & 0.6952 \\ -0.3112 & 0.1511 & 1.7804 & -0.8463 \\ -0.4533 & -0.2798 & -0.0746 & 4.867 \\ -3.9534 & 6.1809 & -14.918 & 25.0205 \\ 18.734 & -35.696 & 81.2939 & -105.6101 \\ 0.8132 & 0.1434 & -0.0019 & 0.1217 \\ -0.0751 & -0.0155 & 0.0429 & 0.226 \\ 0.2353 & -0.125 & -0.9715 & 1.5781 \\ -0.3382 & 0.0772 & 0.9026 & -0.9199 \\ 0.054 & 0.0098 & 0.0683 & 0.0832 \\ 0.2753 & 0.045 & 0.3497 & 0.3819 \\ 0.5106 & -0.08 & -0.6218 & 1.96 \end{pmatrix}$$

## Matrix $S$

$$\begin{array}{c}
 r \\
 C^m \\
 C^h \\
 I \\
 I^m \\
 I^h \\
 K \\
 N \\
 N^m \\
 N^h \\
 U \\
 W \\
 Y
 \end{array}
 \begin{pmatrix}
 \epsilon^h & \epsilon^m \\
 -0.6545 & 2.0631 \\
 -0.9051 & 0.7318 \\
 1.8741 & -0.8908 \\
 -0.0785 & 5.1231 \\
 -15.7031 & 26.3373 \\
 85.5725 & -111.1686 \\
 -0.002 & 0.1281 \\
 0.0452 & 0.2379 \\
 -1.0227 & 1.6612 \\
 0.9501 & -0.9683 \\
 0.0719 & 0.0875 \\
 0.3682 & 0.402 \\
 -0.6545 & 2.0631
 \end{pmatrix}$$

## 10 Model statistics

### 10.1 Basic statistics

	Steady-state value	Std. dev.	Variance	Loglin
$C^m$	0.7224	0.9767	0.954	Y
$C^h$	0.3805	1.52	2.3104	Y
$Y$	1.0367	1.7868	3.1926	Y
$I^m$	0.2658	12.888	166.0993	Y
$I^h$	0.0485	57.2629	3279.0405	Y
$K^m$	10.6329	0.6337	0.4016	Y
$K^h$	1.9397	1.9051	3.6295	Y
$N^m$	0.2799	1.3761	1.8936	Y
$N^h$	0.3303	0.9286	0.8623	Y
$W$	2.3706	0.6601	0.4358	Y

### 10.2 Correlation matrix

	$C^m$	$C^h$	$I^m$	$I^h$	$K^m$	$K^h$	$N^m$	$N^h$	$W$	$Y$
$C^m$	1	-0.843	0.114	0.021	0.842	-0.704	0.781	-0.933	0.056	0.622
$C^h$		1	-0.05	0.084	-0.438	0.657	-0.575	0.833	0.421	-0.288
$I^m$			1	-0.901	0.303	-0.634	0.508	-0.332	0.409	0.542
$I^h$				1	-0.068	0.409	-0.194	0.135	-0.028	-0.16
$K^m$					1	-0.592	0.766	-0.76	0.511	0.779
$K^h$						1	-0.946	0.906	-0.271	-0.829
$N^m$							1	-0.927	0.475	0.946
$N^h$								1	-0.148	-0.768
$W$									1	0.735
$Y$										1



### 10.3 Cross correlations with the reference variable ( $Y$ )

	$\sigma[\cdot]$ rel. to $\sigma[Y]$	$Y_{t-5}$	$Y_{t-4}$	$Y_{t-3}$	$Y_{t-2}$	$Y_{t-1}$	$Y_t$	$Y_{t+1}$	$Y_{t+2}$	$Y_{t+3}$	$Y_{t+4}$	$Y_{t+5}$
$C_t^m$	0.547	-0.194	-0.098	0.033	0.204	0.42	0.622	0.68	0.584	0.483	0.383	0.287
$C_t^h$	0.851	0.044	-0.001	-0.06	-0.136	-0.229	-0.288	-0.318	-0.238	-0.168	-0.108	-0.057
$I_t^m$	7.213	0.145	0.226	0.322	0.433	0.558	0.542	-0.016	-0.08	-0.126	-0.155	-0.172
$I_t^h$	32.048	-0.124	-0.156	-0.192	-0.23	-0.27	-0.16	0.372	0.307	0.246	0.192	0.144
$K_t^m$	0.355	-0.27	-0.149	0.019	0.238	0.516	0.779	0.751	0.692	0.61	0.516	0.416
$K_t^h$	1.066	-0.12	-0.235	-0.373	-0.537	-0.727	-0.829	-0.528	-0.284	-0.092	0.054	0.161
$N_t^m$	0.77	0.026	0.157	0.322	0.522	0.76	0.946	0.75	0.493	0.281	0.11	-0.024
$N_t^h$	0.52	0.04	-0.07	-0.212	-0.387	-0.6	-0.768	-0.677	-0.484	-0.318	-0.179	-0.065
$W_t$	0.369	-0.032	0.068	0.197	0.356	0.547	0.735	0.568	0.415	0.281	0.166	0.071
$Y_t$	1	0.008	0.146	0.32	0.533	0.787	1	0.787	0.533	0.32	0.146	0.008

### 10.4 Autocorrelations

	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5
$C^m$	0.852	0.622	0.418	0.241	0.092
$C^h$	0.771	0.512	0.298	0.124	-0.011
$I^m$	0.076	0.032	-0.003	-0.029	-0.048
$I^h$	-0.078	-0.074	-0.068	-0.061	-0.054
$K^m$	0.868	0.716	0.556	0.398	0.247
$K^h$	0.711	0.467	0.266	0.105	-0.021
$N^m$	0.803	0.537	0.316	0.136	-0.005
$N^h$	0.82	0.555	0.333	0.152	0.008
$W$	0.74	0.511	0.317	0.156	0.026
$Y$	0.787	0.533	0.32	0.146	0.008

### 10.5 Variance decomposition

	$\epsilon^h$	$\epsilon^m$
$C^m$	0.238	0.762
$C^h$	0.616	0.384
$I^m$	0.053	0.947
$I^h$	0.015	0.985
$K^m$	0.198	0.802
$K^h$	0.023	0.977
$N^m$	0.003	0.997
$N^h$	0.106	0.894
$W$	0.796	0.204
$Y$	0.136	0.864

# 11 Impulse response functions

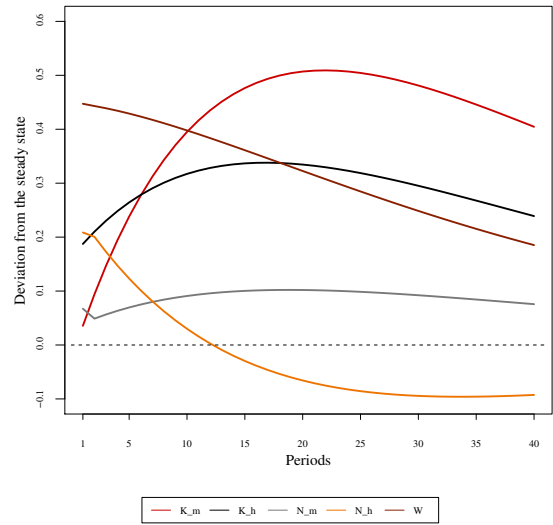
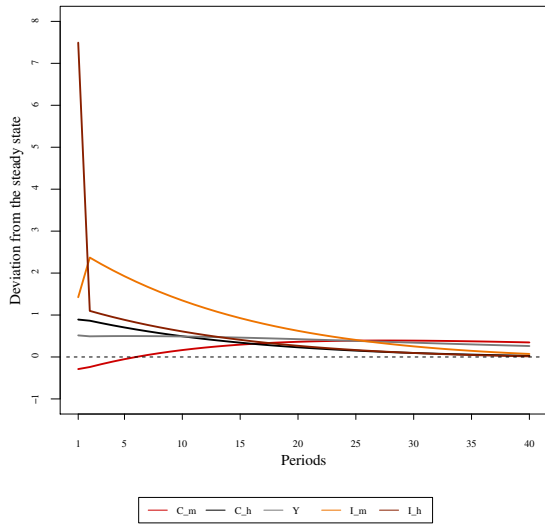


Figure 1: Impulse responses ( $C^m, C^h, Y, I^m, I^h$ ) to  $\epsilon^h$  shock      Figure 2: Impulse responses ( $K^m, K^h, N^m, N^h, W$ ) to  $\epsilon^h$  shock

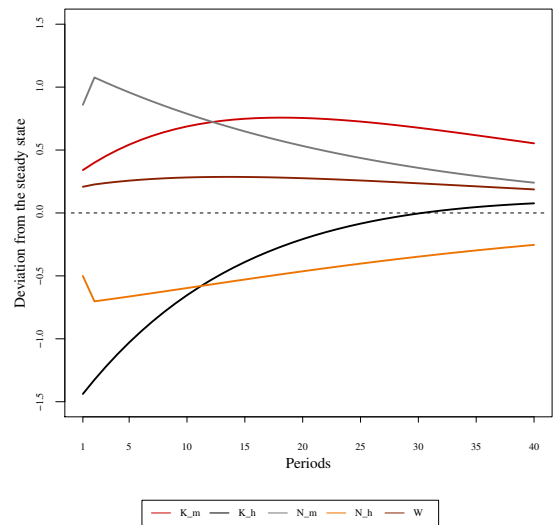
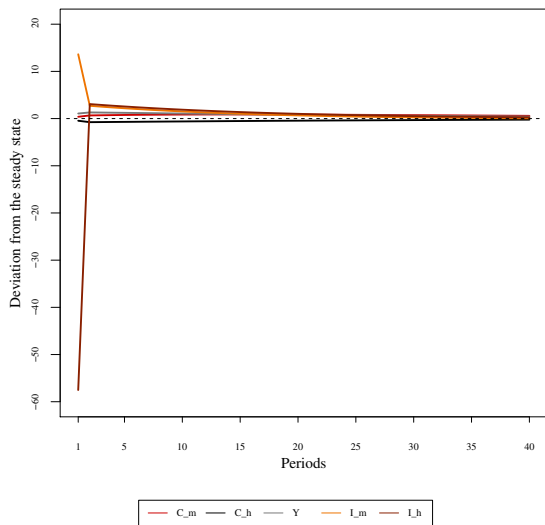


Figure 3: Impulse responses ( $C^m, C^h, Y, I^m, I^h$ ) to  $\epsilon^m$  shock      Figure 4: Impulse responses ( $K^m, K^h, N^m, N^h, W$ ) to  $\epsilon^m$  shock