

Index sets

$$\begin{aligned} agents &= \{A, B\} \\ goods &= \{1, 2, 3\} \end{aligned}$$

1 AGENTS $a \in agents$

1.1 Optimisation problem

$$\max_{(C^{(a,g)})_{g \in goods}} U^{(a)} = \prod_{g \in goods} C^{(a,g)} \alpha^{(a,g)} \quad (1.1)$$

s.t. :

$$\sum_{g \in goods} p^{(g)} C^{(a,g)} = \sum_{g \in goods} e^{(a,g)} p^{(g)} \quad \left(\lambda^{AGENTS^1(a)} \right) \quad (1.2)$$

1.2 Identities

$$g \in goods: \quad e^{(a,g)} = e^{calibr^{(a,g)}} \quad (1.3)$$

1.3 First order conditions

$$g \in goods: \quad -\lambda^{AGENTS^1(a)} p^{(g)} + \alpha^{(a,g)} C^{(a,g)^{-1}} \left(\prod_{g' \in goods} C^{(a,g')} \alpha^{(a,g')} \right) = 0 \quad (C^{(a,g)}) \quad (1.4)$$

2 EQUILIBRIUM

2.1 Identities

$$p^{(1)} = 1 \quad (2.1)$$

$$g \in goods \setminus \{1\}: \quad \sum_{a \in agents} C^{(a,g)} = \sum_{a \in agents} e^{(a,g)} \quad (2.2)$$

3 Equilibrium relationships (before expansion and reduction)

$$1 - p^{(1)} = 0 \quad (3.1)$$

$$a \in agents: \quad U^{(a)} - \prod_{g \in goods} C^{(a,g)} \alpha^{(a,g)} = 0 \quad (3.2)$$

$$a \in agents: \quad \sum_{g \in goods} e^{(a,g)} p^{(g)} - \sum_{g \in goods} p^{(g)} C^{(a,g)} = 0 \quad (3.3)$$

$$a \in agents: \quad g \in goods: \quad e^{calibr^{(a,g)}} - e^{(a,g)} = 0 \quad (3.4)$$

$$a \in agents: \quad g \in goods: \quad -\lambda^{AGENTS^1(a)} p^{(g)} + \alpha^{(a,g)} C^{(a,g)^{-1}} \left(\prod_{g' \in goods} C^{(a,g')} \alpha^{(a,g')} \right) = 0 \quad (3.5)$$

$$g \in goods \setminus \{1\}: \quad \sum_{a \in agents} e^{(a,g)} - \sum_{a \in agents} C^{(a,g)} = 0 \quad (3.6)$$

4 Equilibrium relationships (after expansion and reduction)

$$1 - p^{(1)} = 0 \quad (4.1)$$

$$e^{\text{calibr}\langle A,1 \rangle} - e^{\langle A,1 \rangle} = 0 \quad (4.2)$$

$$e^{\text{calibr}\langle A,2 \rangle} - e^{\langle A,2 \rangle} = 0 \quad (4.3)$$

$$e^{\text{calibr}\langle A,3 \rangle} - e^{\langle A,3 \rangle} = 0 \quad (4.4)$$

$$e^{\text{calibr}\langle B,1 \rangle} - e^{\langle B,1 \rangle} = 0 \quad (4.5)$$

$$e^{\text{calibr}\langle B,2 \rangle} - e^{\langle B,2 \rangle} = 0 \quad (4.6)$$

$$e^{\text{calibr}\langle B,3 \rangle} - e^{\langle B,3 \rangle} = 0 \quad (4.7)$$

$$U^{\langle A \rangle} - C^{\langle A,1 \rangle} \alpha^{\langle A,1 \rangle} C^{\langle A,2 \rangle} \alpha^{\langle A,2 \rangle} C^{\langle A,3 \rangle} \alpha^{\langle A,3 \rangle} = 0 \quad (4.8)$$

$$U^{\langle B \rangle} - C^{\langle B,1 \rangle} \alpha^{\langle B,1 \rangle} C^{\langle B,2 \rangle} \alpha^{\langle B,2 \rangle} C^{\langle B,3 \rangle} \alpha^{\langle B,3 \rangle} = 0 \quad (4.9)$$

$$-\lambda^{\text{AGENTS}^1 \langle A \rangle} p^{(1)} + \alpha^{\langle A,1 \rangle} C^{\langle A,1 \rangle}{}^{-1} C^{\langle A,1 \rangle} \alpha^{\langle A,1 \rangle} C^{\langle A,2 \rangle} \alpha^{\langle A,2 \rangle} C^{\langle A,3 \rangle} \alpha^{\langle A,3 \rangle} = 0 \quad (4.10)$$

$$-\lambda^{\text{AGENTS}^1 \langle A \rangle} p^{(2)} + \alpha^{\langle A,2 \rangle} C^{\langle A,2 \rangle}{}^{-1} C^{\langle A,1 \rangle} \alpha^{\langle A,1 \rangle} C^{\langle A,2 \rangle} \alpha^{\langle A,2 \rangle} C^{\langle A,3 \rangle} \alpha^{\langle A,3 \rangle} = 0 \quad (4.11)$$

$$-\lambda^{\text{AGENTS}^1 \langle A \rangle} p^{(3)} + \alpha^{\langle A,3 \rangle} C^{\langle A,3 \rangle}{}^{-1} C^{\langle A,1 \rangle} \alpha^{\langle A,1 \rangle} C^{\langle A,2 \rangle} \alpha^{\langle A,2 \rangle} C^{\langle A,3 \rangle} \alpha^{\langle A,3 \rangle} = 0 \quad (4.12)$$

$$-\lambda^{\text{AGENTS}^1 \langle B \rangle} p^{(1)} + \alpha^{\langle B,1 \rangle} C^{\langle B,1 \rangle}{}^{-1} C^{\langle B,1 \rangle} \alpha^{\langle B,1 \rangle} C^{\langle B,2 \rangle} \alpha^{\langle B,2 \rangle} C^{\langle B,3 \rangle} \alpha^{\langle B,3 \rangle} = 0 \quad (4.13)$$

$$-\lambda^{\text{AGENTS}^1 \langle B \rangle} p^{(2)} + \alpha^{\langle B,2 \rangle} C^{\langle B,2 \rangle}{}^{-1} C^{\langle B,1 \rangle} \alpha^{\langle B,1 \rangle} C^{\langle B,2 \rangle} \alpha^{\langle B,2 \rangle} C^{\langle B,3 \rangle} \alpha^{\langle B,3 \rangle} = 0 \quad (4.14)$$

$$-\lambda^{\text{AGENTS}^1 \langle B \rangle} p^{(3)} + \alpha^{\langle B,3 \rangle} C^{\langle B,3 \rangle}{}^{-1} C^{\langle B,1 \rangle} \alpha^{\langle B,1 \rangle} C^{\langle B,2 \rangle} \alpha^{\langle B,2 \rangle} C^{\langle B,3 \rangle} \alpha^{\langle B,3 \rangle} = 0 \quad (4.15)$$

$$e^{\langle A,2 \rangle} + e^{\langle B,2 \rangle} - C^{\langle A,2 \rangle} - C^{\langle B,2 \rangle} = 0 \quad (4.16)$$

$$e^{\langle A,3 \rangle} + e^{\langle B,3 \rangle} - C^{\langle A,3 \rangle} - C^{\langle B,3 \rangle} = 0 \quad (4.17)$$

$$e^{\langle A,1 \rangle} p^{(1)} + e^{\langle A,2 \rangle} p^{(2)} + e^{\langle A,3 \rangle} p^{(3)} - p^{(1)} C^{\langle A,1 \rangle} - p^{(2)} C^{\langle A,2 \rangle} - p^{(3)} C^{\langle A,3 \rangle} = 0 \quad (4.18)$$

$$e^{\langle B,1 \rangle} p^{(1)} + e^{\langle B,2 \rangle} p^{(2)} + e^{\langle B,3 \rangle} p^{(3)} - p^{(1)} C^{\langle B,1 \rangle} - p^{(2)} C^{\langle B,2 \rangle} - p^{(3)} C^{\langle B,3 \rangle} = 0 \quad (4.19)$$

5 Equilibrium values

	Equilibrium value
$e^{\langle A,1 \rangle}$	3
$e^{\langle A,2 \rangle}$	2
$e^{\langle A,3 \rangle}$	1
$\lambda^{\text{AGENTS}^1 \langle A \rangle}$	0.2674
$C^{\langle A,1 \rangle}$	2.2667
$C^{\langle A,2 \rangle}$	1.7
$C^{\langle A,3 \rangle}$	2.2667
$U^{\langle A \rangle}$	2.0203

6 Equilibrium values

	Equilibrium value
$e^{(B,1)}$	1
$e^{(B,2)}$	1
$e^{(B,3)}$	3
$\lambda^{\text{AGENTS}^{1(B)}}$	0.2674
$C^{(B,1)}$	1.7333
$C^{(B,2)}$	1.3
$C^{(B,3)}$	1.7333
$U^{(B)}$	1.5449