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1 CONSUMER

1.1 Optimisation problem

$$\max_{K^{\mathrm{s}}_t, C_t, L^{\mathrm{s}}_t, I_t} U_t = \beta \mathrm{E}_t \left[U_{t+1}\right] + (1-\eta)^{-1} \left({C_t}^{\mu} (1 - L^{\mathrm{s}}_t)^{1-\mu}\right)^{1-\eta} \tag{1.1}$$

s.t. :

$$C_t + I_t = \pi_t + K^{\mathrm{s}}_{t-1} r_t + L^{\mathrm{s}}_t W_t \quad \left(\lambda^{\mathrm{CONSUMER}^1}_t\right) \tag{1.2}$$

$$K^{\mathrm{s}}_t = I_t + K^{\mathrm{s}}_{t-1} (1-\delta) \quad \left(\lambda^{\mathrm{CONSUMER}^2}_t\right) \tag{1.3}$$

1.2 First order conditions

$$-\lambda^{\mathrm{CONSUMER}^2}_t + \beta \left((1-\delta) \mathrm{E}_t \left[\lambda^{\mathrm{CONSUMER}^2}_{t+1}\right] + \mathrm{E}_t \left[\lambda^{\mathrm{CONSUMER}^1}_{t+1} r_{t+1}\right]\right) = 0 \quad (K^{\mathrm{s}}_t) \tag{1.4}$$

$$-\lambda^{\mathrm{CONSUMER}^1}_t + \mu {C_t}^{-1+\mu} (1 - L^{\mathrm{s}}_t)^{1-\mu} \left({C_t}^{\mu} (1 - L^{\mathrm{s}}_t)^{1-\mu}\right)^{-\eta} = 0 \quad (C_t) \tag{1.5}$$

$$\lambda^{\mathrm{CONSUMER}^1}_t W_t + (-1+\mu) {C_t}^{\mu} (1 - L^{\mathrm{s}}_t)^{-\mu} \left({C_t}^{\mu} (1 - L^{\mathrm{s}}_t)^{1-\mu}\right)^{-\eta} = 0 \quad (L^{\mathrm{s}}_t) \tag{1.6}$$

$$-\lambda^{\mathrm{CONSUMER}^1}_t + \lambda^{\mathrm{CONSUMER}^2}_t = 0 \quad (I_t) \tag{1.7}$$

2 FIRM

2.1 Optimisation problem

$$\max_{K^{\mathrm{d}}_t, L^{\mathrm{d}}_t, Y_t} \pi_t = Y_t - L^{\mathrm{d}}_t W_t - r_t K^{\mathrm{d}}_t \tag{2.1}$$

s.t. :

$$Y_t = Z_t {K^{\mathrm{d}}_t}^{\alpha} {L^{\mathrm{d}}_t}^{1-\alpha} \quad \left(\lambda^{\mathrm{FIRM}^1}_t\right) \tag{2.2}$$

2.2 First order conditions

$$-r_t + \alpha \lambda_t^{\mathrm{FIRM}^1} Z_t {K_t^{\mathrm{d}}}^{-1+\alpha} {L_t^{\mathrm{d}}}^{1-\alpha} = 0 \quad \left(K_t^{\mathrm{d}}\right) \tag{2.3}$$

$$-W_t + \lambda_t^{\mathrm{FIRM}^1} Z_t \left(1-\alpha\right) {K_t^{\mathrm{d}}}^{\alpha} {L_t^{\mathrm{d}}}^{-\alpha} = 0 \quad \left(L_t^{\mathrm{d}}\right) \tag{2.4}$$

$$1 - \lambda_t^{\mathrm{FIRM}^1} = 0 \quad \left(Y_t\right) \tag{2.5}$$

2.3 First order conditions after reduction

$$-r_t + \alpha Z_t {K_t^{\mathrm{d}}}^{-1+\alpha} {L_t^{\mathrm{d}}}^{1-\alpha} = 0 \quad \left(K_t^{\mathrm{d}}\right) \tag{2.6}$$

$$-W_t + Z_t \left(1-\alpha\right) {K_t^{\mathrm{d}}}^{\alpha} {L_t^{\mathrm{d}}}^{-\alpha} = 0 \quad \left(L_t^{\mathrm{d}}\right) \tag{2.7}$$

3 EQUILIBRIUM

3.1 Identities

$$K_t^{\mathrm{d}} = K_{t-1}^{\mathrm{s}} \tag{3.1}$$

$$L_t^{\mathrm{d}} = L_t^{\mathrm{s}} \tag{3.2}$$

4 EXOG

4.1 Identities

$$Z_t = e^{\epsilon_t^{\mathrm{Z}} + \phi \log Z_{t-1}} \tag{4.1}$$

5 Equilibrium relationships (after reduction)

$$-r_t + \alpha Z_t {K_{t-1}^{\mathrm{s}}}^{-1+\alpha} {L_t^{\mathrm{s}}}^{1-\alpha} = 0 \tag{5.1}$$

$$-W_t + Z_t \left(1-\alpha\right) {K_{t-1}^{\mathrm{s}}}^{\alpha} {L_t^{\mathrm{s}}}^{-\alpha} = 0 \tag{5.2}$$

$$-Y_t + Z_t {K_{t-1}^{\mathrm{s}}}^{\alpha} {L_t^{\mathrm{s}}}^{1-\alpha} = 0 \tag{5.3}$$

$$-Z_t + e^{\epsilon_t^{\mathrm{Z}} + \phi \log Z_{t-1}} = 0 \tag{5.4}$$

$$\beta \left(\mu \mathrm{E}_t \left[r_{t+1} {C_{t+1}}^{-1+\mu} \left(1 - L_{t+1}^{\mathrm{s}}\right)^{1-\mu} \left({C_{t+1}}^{\mu} \left(1 - L_{t+1}^{\mathrm{s}}\right)^{1-\mu}\right)^{-\eta}\right] + \mu \left(1-\delta\right) \mathrm{E}_t \left[{C_{t+1}}^{-1+\mu} \left(1 - L_{t+1}^{\mathrm{s}}\right)^{1-\mu} \left({C_{t+1}}^{\mu} \left(1 - L_{t+1}^{\mathrm{s}}\right)^{1-\mu}\right)^{-\eta}\right]\right) - \mu {C_t}^{-1+\mu} \left(1 - L_t^{\mathrm{s}}\right)^{1-\mu} \left({C_t}^{\mu} \left(1 - L_t^{\mathrm{s}}\right)^{1-\mu}\right)^{-\eta} = \tag{5.5}$$

$$\left(-1+\mu\right) {C_t}^{\mu} \left(1 - L_t^{\mathrm{s}}\right)^{-\mu} \left({C_t}^{\mu} \left(1 - L_t^{\mathrm{s}}\right)^{1-\mu}\right)^{-\eta} + \mu W_t {C_t}^{-1+\mu} \left(1 - L_t^{\mathrm{s}}\right)^{1-\mu} \left({C_t}^{\mu} \left(1 - L_t^{\mathrm{s}}\right)^{1-\mu}\right)^{-\eta} = 0 \tag{5.6}$$

$$-C_t - I_t + Y_t = 0 \tag{5.7}$$

$$I_t - K^{\mathrm{s}}_t + K^{\mathrm{s}}_{t-1}\left(1-\delta\right) = 0 \tag{5.8}$$

$$U_t - \beta \mathrm{E}_t\left[U_{t+1}\right] - \left(1-\eta\right)^{-1}\left({C_t}^{\mu}(1-L^{\mathrm{s}}_t)^{1-\mu}\right)^{1-\eta} = 0 \tag{5.9}$$

6 Steady state relationships (after reduction)

$$-r_{\mathrm{ss}} + \alpha Z_{\mathrm{ss}} {K^{\mathrm{s}}_{\mathrm{ss}}}^{-1+\alpha} {L^{\mathrm{s}}_{\mathrm{ss}}}^{1-\alpha} = 0 \tag{6.1}$$

$$-W_{\mathrm{ss}} + Z_{\mathrm{ss}}\left(1-\alpha\right) {K^{\mathrm{s}}_{\mathrm{ss}}}^{\alpha} {L^{\mathrm{s}}_{\mathrm{ss}}}^{-\alpha} = 0 \tag{6.2}$$

$$-Y_{\mathrm{ss}} + Z_{\mathrm{ss}} {K^{\mathrm{s}}_{\mathrm{ss}}}^{\alpha} {L^{\mathrm{s}}_{\mathrm{ss}}}^{1-\alpha} = 0 \tag{6.3}$$

$$-Z_{\mathrm{ss}} + e^{\phi \log Z_{\mathrm{ss}}} = 0 \tag{6.4}$$

$$\beta\left(\mu r_{\mathrm{ss}} {C_{\mathrm{ss}}}^{-1+\mu}(1-L^{\mathrm{s}}_{\mathrm{ss}})^{1-\mu}\left({C_{\mathrm{ss}}}^{\mu}(1-L^{\mathrm{s}}_{\mathrm{ss}})^{1-\mu}\right)^{-\eta} + \mu\left(1-\delta\right) {C_{\mathrm{ss}}}^{-1+\mu}(1-L^{\mathrm{s}}_{\mathrm{ss}})^{1-\mu}\left({C_{\mathrm{ss}}}^{\mu}(1-L^{\mathrm{s}}_{\mathrm{ss}})^{1-\mu}\right)^{-\eta}\right) - \mu {C_{\mathrm{ss}}}^{-1+\mu}(1-L^{\mathrm{s}}_{\mathrm{ss}})^{1-\mu}\left({C_{\mathrm{ss}}}^{\mu}(1-L^{\mathrm{s}}_{\mathrm{ss}})^{1-\mu}\right)^{-\eta} = 0 \tag{6.5}$$

$$\left(-1+\mu\right) {C_{\mathrm{ss}}}^{\mu}(1-L^{\mathrm{s}}_{\mathrm{ss}})^{-\mu}\left({C_{\mathrm{ss}}}^{\mu}(1-L^{\mathrm{s}}_{\mathrm{ss}})^{1-\mu}\right)^{-\eta} + \mu W_{\mathrm{ss}} {C_{\mathrm{ss}}}^{-1+\mu}(1-L^{\mathrm{s}}_{\mathrm{ss}})^{1-\mu}\left({C_{\mathrm{ss}}}^{\mu}(1-L^{\mathrm{s}}_{\mathrm{ss}})^{1-\mu}\right)^{-\eta} = 0 \tag{6.6}$$

$$-C_{\mathrm{ss}} - I_{\mathrm{ss}} + Y_{\mathrm{ss}} = 0 \tag{6.7}$$

$$I_{\mathrm{ss}} - K^{\mathrm{s}}_{\mathrm{ss}} + K^{\mathrm{s}}_{\mathrm{ss}}\left(1-\delta\right) = 0 \tag{6.8}$$

$$U_{\mathrm{ss}} - \beta U_{\mathrm{ss}} - \left(1-\eta\right)^{-1}\left({C_{\mathrm{ss}}}^{\mu}(1-L^{\mathrm{s}}_{\mathrm{ss}})^{1-\mu}\right)^{1-\eta} = 0 \tag{6.9}$$

7 Calibrating equations

$$-0.36 Y_{\mathrm{ss}} + r_{\mathrm{ss}} K^{\mathrm{s}}_{\mathrm{ss}} = 0 \tag{7.1}$$

8 Parameter settings

$$\beta = 0.99 \tag{8.1}$$

$$\delta = 0.025 \tag{8.2}$$

$$\eta = 2 \tag{8.3}$$

$$\mu = 0.3 \tag{8.4}$$

$$\phi = 0.95 \tag{8.5}$$

9 Steady-state values

	Steady-state value
r	0.0351
C	0.7422
I	0.2559
K^{s}	10.2368
L^{s}	0.2695
U	-136.2372
W	2.3706
Y	0.9981
Z	1

10 The solution of the 1st order perturbation

Matrix P

$$\begin{array}{c c} & \begin{array}{cc} K^{\mathrm{s}}_{t-1} & Z_{t-1} \end{array} \\ \begin{array}{c} K^{\mathrm{s}}_t \\ Z_t \end{array} & \begin{pmatrix} 0.9631 & 0.0962 \\ 0 & 0.95 \end{pmatrix} \end{array}$$

Matrix Q

$$\begin{array}{c c} & \epsilon^{\mathrm{Z}} \\ \begin{array}{c} K^{\mathrm{s}} \\ Z \end{array} & \begin{pmatrix} 0.1012 \\ 1 \end{pmatrix} \end{array}$$

Matrix R

$$\begin{array}{c c} & \begin{array}{cc} K^{\mathrm{s}}_{t-1} & Z_{t-1} \end{array} \\ \begin{array}{c} r_t \\ C_t \\ I_t \\ L^{\mathrm{s}}_t \\ U_t \\ W_t \\ Y_t \end{array} & \begin{pmatrix} -0.7559 & 1.3521 \\ 0.4919 & 0.4921 \\ -0.4745 & 3.8461 \\ -0.181 & 0.6282 \\ 0.0418 & 0.0644 \\ 0.4252 & 0.7238 \\ 0.2441 & 1.3521 \end{pmatrix} \end{array}$$

Matrix S

$$\begin{array}{c c} & \epsilon^{\mathrm{Z}} \\ \begin{array}{c} r \\ C \\ I \\ L^{\mathrm{s}} \\ U \\ W \\ Y \end{array} & \begin{pmatrix} 1.4232 \\ 0.518 \\ 4.0485 \\ 0.6613 \\ 0.0678 \\ 0.7619 \\ 1.4232 \end{pmatrix} \end{array}$$

11 Model statistics

11.1 Basic statistics

	Steady-state value	Std. dev.	Variance	Loglin
r	0.0351	0.1893	0.0358	Y
C	0.7422	0.0711	0.0051	Y
I	0.2559	0.5284	0.2792	Y
K^{s}	10.2368	0.0469	0.0022	Y
L^{s}	0.2695	0.0867	0.0075	Y
U	-136.2372	0.009	0.0001	Y
W	2.3706	0.1011	0.0102	Y
Y	0.9981	0.1857	0.0345	Y
Z	1	0.1303	0.017	Y

11.2 Correlation matrix

	r	C	I	K^{s}	L^{s}	U	W	Y	Z
r	1	0.868	0.989	0.084	0.996	0.918	0.926	0.969	0.982
C		1	0.931	0.567	0.909	0.994	0.991	0.964	0.946
I			1	0.228	0.998	0.966	0.971	0.995	0.999
K^{s}				1	0.174	0.472	0.454	0.328	0.269
L^{s}					1	0.95	0.956	0.987	0.995
U						1	1	0.988	0.976
W							1	0.991	0.98
Y								1	0.998
Z									1

11.3 Cross correlations with the reference variable (Y)

	$\sigma[\cdot]$ rel. to $\sigma[Y]$	Y_{t-5}	Y_{t-4}	Y_{t-3}	Y_{t-2}	Y_{t-1}	Y_t	Y_{t+1}	Y_{t+2}	Y_{t+3}	Y_{t+4}	Y_{t+5}
r_t	1.019	0.116	0.234	0.378	0.548	0.745	0.969	0.623	0.343	0.121	-0.048	-0.172
C_t	0.383	-0.14	-0.013	0.156	0.372	0.64	0.964	0.77	0.592	0.431	0.29	0.169
I_t	2.845	0.044	0.168	0.323	0.511	0.735	0.995	0.684	0.426	0.216	0.05	-0.077
K^{s}_t	0.253	-0.474	-0.415	-0.313	-0.162	0.049	0.328	0.513	0.62	0.665	0.662	0.624
L^{s}_t	0.467	0.072	0.194	0.345	0.527	0.741	0.987	0.662	0.395	0.18	0.013	-0.113
U_t	0.049	-0.086	0.042	0.21	0.42	0.679	0.988	0.755	0.55	0.372	0.221	0.096
W_t	0.544	-0.076	0.052	0.219	0.428	0.685	0.991	0.752	0.541	0.361	0.208	0.083
Y_t	1	-0.008	0.119	0.28	0.479	0.718	1	0.718	0.479	0.28	0.119	-0.008
Z_t	0.702	0.023	0.149	0.306	0.499	0.729	0.998	0.699	0.448	0.242	0.078	-0.049

11.4 Autocorrelations

	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5
r	0.71	0.466	0.265	0.103	-0.022
C	0.76	0.545	0.357	0.196	0.063
I	0.711	0.467	0.267	0.105	-0.02
K^{s}	0.96	0.862	0.727	0.571	0.407
L^{s}	0.709	0.465	0.264	0.102	-0.023
U	0.739	0.512	0.319	0.158	0.028
W	0.736	0.507	0.312	0.152	0.022
Y	0.718	0.479	0.28	0.119	-0.008
Z	0.713	0.471	0.271	0.11	-0.016

12 Impulse response functions

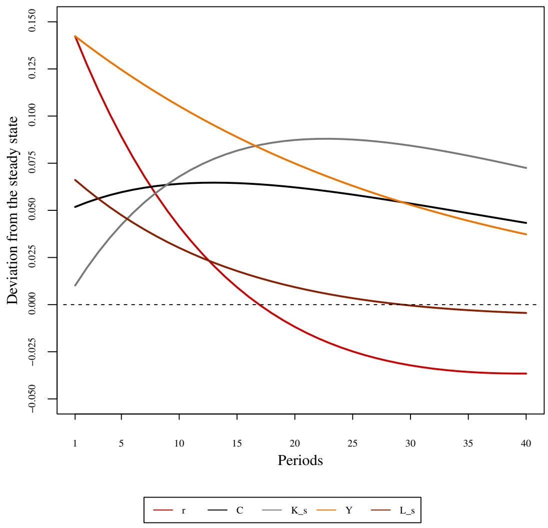

Figure 1: Impulse responses $(r, C, K^{\mathrm{s}}, Y, L^{\mathrm{s}})$ to ϵ^{Z} shock

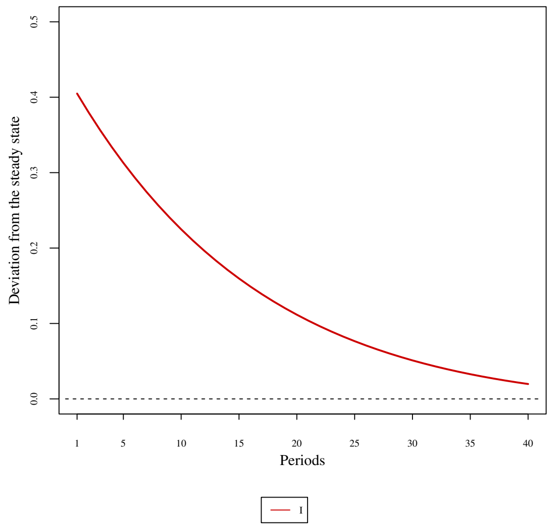

Figure 2: Impulse response (I) to ϵ^{Z} shock