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1 CONSUMER

1.1 Optimisation problem

$$\max_{C_t, L^{\mathrm{s}}_t} U_t = \beta \mathrm{E}_t\left[U_{t+1}\right] + (1-\eta)^{-1}\left({C_t}^{\mu}(1-L^{\mathrm{s}}_t)^{1-\mu}\right)^{1-\eta} \tag{1.1}$$

s.t. :

$$C_t = \pi_t + L^{\mathrm{s}}_t W_t \quad (\lambda^{\mathrm{c}}_t) \tag{1.2}$$

1.2 First order conditions

$$\beta - \lambda^{\mathrm{U}}_t = 0 \quad (U_t) \tag{1.3}$$

$$-\lambda^{\mathrm{c}}_t + \mu {C_t}^{-1+\mu}(1-L^{\mathrm{s}}_t)^{1-\mu}\left({C_t}^{\mu}(1-L^{\mathrm{s}}_t)^{1-\mu}\right)^{-\eta} = 0 \quad (C_t) \tag{1.4}$$

$$\lambda^{\mathrm{c}}_t W_t + (-1+\mu)\, {C_t}^{\mu}(1-L^{\mathrm{s}}_t)^{-\mu}\left({C_t}^{\mu}(1-L^{\mathrm{s}}_t)^{1-\mu}\right)^{-\eta} = 0 \quad (L^{\mathrm{s}}_t) \tag{1.5}$$

2 FIRM

2.1 Optimisation problem

$$\max_{K_t, L^{\mathrm{d}}_t, Y_t, I_t, \pi_t, CapUt_t} \Pi_t = \pi_t + {\lambda^{\mathrm{c}}_t}^{-1}\mathrm{E}_t\left[\lambda^{\mathrm{c}}_{t+1}\lambda^{\mathrm{U}}_{t+1}\Pi_{t+1}\right] \tag{2.1}$$

s.t. :

$$Y_t = {L^{\mathrm{d}}_t}^{1-\alpha}{Z_t}^{1-\alpha}(K_{t-1} CapUt_t)^{\alpha} \quad \left(\lambda^{\mathrm{FIRM}^1}_t\right) \tag{2.2}$$

$$K_t = I_t + K_{t-1}\left(1 - \delta {CapUt_t}^{\omega}\right) \quad \left(\lambda^{\mathrm{FIRM}^2}_t\right) \tag{2.3}$$

$$\pi_t = -I_t - L^{\mathrm{d}}_t W_t + P_t Y_t \quad \left(\lambda^{\mathrm{FIRM}^3}_t\right) \tag{2.4}$$

2.2 First order conditions

$$-\lambda^{\mathrm{FIRM}^\Pi}_t + {\lambda^{\mathrm{c}}_{t-1}}^{-1}\lambda^{\mathrm{c}}_t\lambda^{\mathrm{U}}_t = 0 \quad (\Pi_t) \tag{2.5}$$

$$-\lambda^{\mathrm{FIRM}^2}_t + \mathrm{E}_t\left[\lambda^{\mathrm{FIRM}^\Pi}_{t+1}\left(\lambda^{\mathrm{FIRM}^2}_{t+1}\left(1 - \delta {CapUt_{t+1}}^{\omega}\right) + \alpha\lambda^{\mathrm{FIRM}^1}_{t+1} CapUt_{t+1} {L^{\mathrm{d}}_{t+1}}^{1-\alpha}{Z_{t+1}}^{1-\alpha}\left(K_t CapUt_{t+1}\right)^{-1+\alpha}\right)\right] = 0 \quad (K_t) \tag{2.6}$$

$$-\lambda_t^{\mathrm{FIRM}^3} W_t + \lambda_t^{\mathrm{FIRM}^1} \left(1-\alpha\right) {L^{\mathrm{d}}_t}^{-\alpha} {Z_t}^{1-\alpha} \left(K_{t-1} \mathit{CapUt}_t\right)^{\alpha} = 0 \quad \left(L^{\mathrm{d}}_t\right) \tag{2.7}$$

$$-\lambda_t^{\mathrm{FIRM}^1} + \lambda_t^{\mathrm{FIRM}^3} P_t = 0 \quad (Y_t) \tag{2.8}$$

$$\lambda_t^{\mathrm{FIRM}^2} - \lambda_t^{\mathrm{FIRM}^3} = 0 \quad (I_t) \tag{2.9}$$

$$1 - \lambda_t^{\mathrm{FIRM}^3} = 0 \quad (\pi_t) \tag{2.10}$$

$$-\delta \omega K_{t-1} \lambda_t^{\mathrm{FIRM}^2} {\mathit{CapUt}_t}^{-1+\omega} + \alpha K_{t-1} \lambda_t^{\mathrm{FIRM}^1} {L^{\mathrm{d}}_t}^{1-\alpha} {Z_t}^{1-\alpha} \left(K_{t-1} \mathit{CapUt}_t\right)^{-1+\alpha} = 0 \quad (\mathit{CapUt}_t) \tag{2.11}$$

2.3 First order conditions after reduction

$$-\lambda_t^{\mathrm{FIRM}^\Pi} + {\lambda^{\mathrm{c}}_{t-1}}^{-1} \lambda^{\mathrm{c}}_t \lambda^{\mathrm{U}}_t = 0 \quad (\Pi_t) \tag{2.12}$$

$$-1 + \mathrm{E}_t \left[\lambda_{t+1}^{\mathrm{FIRM}^\Pi} \left(1 - \delta {\mathit{CapUt}_{t+1}}^{\omega} + \alpha \lambda_{t+1}^{\mathrm{FIRM}^1} \mathit{CapUt}_{t+1} {L^{\mathrm{d}}_{t+1}}^{1-\alpha} {Z_{t+1}}^{1-\alpha} \left(K_t \mathit{CapUt}_{t+1}\right)^{-1+\alpha}\right)\right] = 0 \quad (K_t) \tag{2.13}$$

$$-W_t + \lambda_t^{\mathrm{FIRM}^1} \left(1-\alpha\right) {L^{\mathrm{d}}_t}^{-\alpha} {Z_t}^{1-\alpha} \left(K_{t-1} \mathit{CapUt}_t\right)^{\alpha} = 0 \quad \left(L^{\mathrm{d}}_t\right) \tag{2.14}$$

$$-\lambda_t^{\mathrm{FIRM}^1} + P_t = 0 \quad (Y_t) \tag{2.15}$$

$$-\delta \omega K_{t-1} {\mathit{CapUt}_t}^{-1+\omega} + \alpha K_{t-1} \lambda_t^{\mathrm{FIRM}^1} {L^{\mathrm{d}}_t}^{1-\alpha} {Z_t}^{1-\alpha} \left(K_{t-1} \mathit{CapUt}_t\right)^{-1+\alpha} = 0 \quad (\mathit{CapUt}_t) \tag{2.16}$$

3 EQUILIBRIUM

3.1 Identities

$$P_t = 1 \tag{3.1}$$

$$L^{\mathrm{d}}_t = L^{\mathrm{s}}_t \tag{3.2}$$

4 EXOG

4.1 Identities

$$Z_t = e^{\epsilon^{\mathrm{Z}}_t + \phi \log Z_{t-1}} \tag{4.1}$$

5 Equilibrium relationships (after reduction)

$$-1+\beta {C_t}^{1-\mu}(1-L^{\mathrm{s}}_t)^{-1+\mu}\left({C_t}^{\mu}(1-L^{\mathrm{s}}_t)^{1-\mu}\right)^{\eta}\mathrm{E}_t\left[\left(1-\delta {\mathit{CapUt}_{t+1}}^{\omega}+\alpha \mathit{CapUt}_{t+1}{L^{\mathrm{s}}_{t+1}}^{1-\alpha}{Z_{t+1}}^{1-\alpha}\left(K_t \mathit{CapUt}_{t+1}\right)^{-1+\alpha}\right){C_{t+1}}^{-1+\mu}\left(1-L^{\mathrm{s}}_{t+1}\right)^{1-\mu}\left({C_{t+1}}^{\mu}\left(1-L^{\mathrm{s}}_{t+1}\right)^{1-\mu}\right)^{-\eta}\right]=0 \tag{5.1}$$

$$-W_t+(1-\alpha)\,L^{\mathrm{s}\,-\alpha}_t {Z_t}^{1-\alpha}(K_{t-1}\mathit{CapUt}_t)^{\alpha}=0 \tag{5.2}$$

$$-Y_t+L^{\mathrm{s}\,1-\alpha}_t {Z_t}^{1-\alpha}(K_{t-1}\mathit{CapUt}_t)^{\alpha}=0 \tag{5.3}$$

$$-Z_t+e^{\epsilon^{\mathrm{Z}}_t+\phi\log Z_{t-1}}=0 \tag{5.4}$$

$$-\delta\omega K_{t-1}{\mathit{CapUt}_t}^{-1+\omega}+\alpha K_{t-1}L^{\mathrm{s}\,1-\alpha}_t {Z_t}^{1-\alpha}(K_{t-1}\mathit{CapUt}_t)^{-1+\alpha}=0 \tag{5.5}$$

$$(-1+\mu)\,{C_t}^{\mu}(1-L^{\mathrm{s}}_t)^{-\mu}\left({C_t}^{\mu}(1-L^{\mathrm{s}}_t)^{1-\mu}\right)^{-\eta}+\mu W_t {C_t}^{-1+\mu}(1-L^{\mathrm{s}}_t)^{1-\mu}\left({C_t}^{\mu}(1-L^{\mathrm{s}}_t)^{1-\mu}\right)^{-\eta}=0 \tag{5.6}$$

$$I_t-K_t+K_{t-1}\left(1-\delta {\mathit{CapUt}_t}^{\omega}\right)=0 \tag{5.7}$$

$$U_t-\beta\mathrm{E}_t\left[U_{t+1}\right]-(1-\eta)^{-1}\left({C_t}^{\mu}(1-L^{\mathrm{s}}_t)^{1-\mu}\right)^{1-\eta}=0 \tag{5.8}$$

$$-C_t+\Pi_t+L^{\mathrm{s}}_t W_t-\beta\left({C_t}^{-1+\mu}\right)^{-1}\left((1-L^{\mathrm{s}}_t)^{1-\mu}\right)^{-1}\left(\left({C_t}^{\mu}(1-L^{\mathrm{s}}_t)^{1-\mu}\right)^{-\eta}\right)^{-1}\mathrm{E}_t\left[\Pi_{t+1}{C_{t+1}}^{-1+\mu}\left(1-L^{\mathrm{s}}_{t+1}\right)^{1-\mu}\left({C_{t+1}}^{\mu}\left(1-L^{\mathrm{s}}_{t+1}\right)^{1-\mu}\right)^{-\eta}\right]=0 \tag{5.9}$$

$$-I_t-\Pi_t+Y_t-L^{\mathrm{s}}_t W_t+\beta\left({C_t}^{-1+\mu}\right)^{-1}\left((1-L^{\mathrm{s}}_t)^{1-\mu}\right)^{-1}\left(\left({C_t}^{\mu}(1-L^{\mathrm{s}}_t)^{1-\mu}\right)^{-\eta}\right)^{-1}\mathrm{E}_t\left[\Pi_{t+1}{C_{t+1}}^{-1+\mu}\left(1-L^{\mathrm{s}}_{t+1}\right)^{1-\mu}\left({C_{t+1}}^{\mu}\left(1-L^{\mathrm{s}}_{t+1}\right)^{1-\mu}\right)^{-\eta}\right]=0 \tag{5.10}$$

6 Steady state relationships (after reduction)

$$-1+\beta\left(1-\delta {\mathit{CapUt}_{\mathrm{ss}}}^{\omega}+\alpha \mathit{CapUt}_{\mathrm{ss}}{L^{\mathrm{s}}_{\mathrm{ss}}}^{1-\alpha}{Z_{\mathrm{ss}}}^{1-\alpha}(\mathit{CapUt}_{\mathrm{ss}}K_{\mathrm{ss}})^{-1+\alpha}\right){C_{\mathrm{ss}}}^{-1+\mu}{C_{\mathrm{ss}}}^{1-\mu}(1-L^{\mathrm{s}}_{\mathrm{ss}})^{-1+\mu}(1-L^{\mathrm{s}}_{\mathrm{ss}})^{1-\mu}=0 \tag{6.1}$$

$$-W_{\mathrm{ss}}+(1-\alpha)\,{L^{\mathrm{s}}_{\mathrm{ss}}}^{-\alpha}{Z_{\mathrm{ss}}}^{1-\alpha}(\mathit{CapUt}_{\mathrm{ss}}K_{\mathrm{ss}})^{\alpha}=0 \tag{6.2}$$

$$-Y_{\mathrm{ss}}+{L^{\mathrm{s}}_{\mathrm{ss}}}^{1-\alpha}{Z_{\mathrm{ss}}}^{1-\alpha}(\mathit{CapUt}_{\mathrm{ss}}K_{\mathrm{ss}})^{\alpha}=0 \tag{6.3}$$

$$-Z_{\mathrm{ss}}+e^{\phi\log Z_{\mathrm{ss}}}=0 \tag{6.4}$$

$$-\delta\omega K_{\mathrm{ss}}{\mathit{CapUt}_{\mathrm{ss}}}^{-1+\omega}+\alpha K_{\mathrm{ss}}{L^{\mathrm{s}}_{\mathrm{ss}}}^{1-\alpha}{Z_{\mathrm{ss}}}^{1-\alpha}(\mathit{CapUt}_{\mathrm{ss}}K_{\mathrm{ss}})^{-1+\alpha}=0 \tag{6.5}$$

$$(-1+\mu)\,{C_{\mathrm{ss}}}^{\mu}(1-L^{\mathrm{s}}_{\mathrm{ss}})^{-\mu}\left({C_{\mathrm{ss}}}^{\mu}(1-L^{\mathrm{s}}_{\mathrm{ss}})^{1-\mu}\right)^{-\eta}+\mu W_{\mathrm{ss}}{C_{\mathrm{ss}}}^{-1+\mu}(1-L^{\mathrm{s}}_{\mathrm{ss}})^{1-\mu}\left({C_{\mathrm{ss}}}^{\mu}(1-L^{\mathrm{s}}_{\mathrm{ss}})^{1-\mu}\right)^{-\eta}=0 \tag{6.6}$$

$$I_{\mathrm{ss}}-K_{\mathrm{ss}}+K_{\mathrm{ss}}\left(1-\delta {\mathit{CapUt}_{\mathrm{ss}}}^{\omega}\right)=0 \tag{6.7}$$

$$U_{\mathrm{ss}}-\beta U_{\mathrm{ss}}-(1-\eta)^{-1}\left({C_{\mathrm{ss}}}^{\mu}(1-L^{\mathrm{s}}_{\mathrm{ss}})^{1-\mu}\right)^{1-\eta}=0 \tag{6.8}$$

$$-C_{\mathrm{ss}}+\Pi_{\mathrm{ss}}+L^{\mathrm{s}}_{\mathrm{ss}}W_{\mathrm{ss}}-\beta\Pi_{\mathrm{ss}}1(1-L^{\mathrm{s}}_{\mathrm{ss}})^{-1+\mu}(1-L^{\mathrm{s}}_{\mathrm{ss}})^{1-\mu}=0 \tag{6.9}$$

$$-I_{\mathrm{ss}}-\Pi_{\mathrm{ss}}+Y_{\mathrm{ss}}-L^{\mathrm{s}}_{\mathrm{ss}}W_{\mathrm{ss}}+\beta\Pi_{\mathrm{ss}}{C_{\mathrm{ss}}}^{-1+\mu}{C_{\mathrm{ss}}}^{1-\mu}(1-L^{\mathrm{s}}_{\mathrm{ss}})^{-1+\mu}(1-L^{\mathrm{s}}_{\mathrm{ss}})^{1-\mu}=0 \tag{6.10}$$

7 Parameter settings

$$\alpha = 0.36 \tag{7.1}$$

$$\beta = 0.99 \tag{7.2}$$

$$\delta = 0.025 \tag{7.3}$$

$$\eta = 2 \tag{7.4}$$

$$\mu = 0.3 \tag{7.5}$$

$$\omega = 1.45 \tag{7.6}$$

$$\phi = 0.95 \tag{7.7}$$

8 Steady-state values

	Steady-state value
C	0.7449
$CapUt$	0.9284
I	0.246
K	10.96
L^{s}	0.2673
Π	11.0707
U	-135.8123
W	2.3722
Y	0.9909
Z	1

9 The solution of the 1st order perturbation

Matrix P

$$\begin{array}{c} \\ K_t \\ Z_t \end{array} \begin{array}{c} \begin{array}{cc} K_{t-1} & Z_{t-1} \end{array} \\ \begin{pmatrix} 0.9758 & 0.0705 \\ 0 & 0.95 \end{pmatrix} \end{array}$$

Matrix Q

$$\begin{array}{c} \\ K \\ Z \end{array} \begin{array}{c} \epsilon^{\mathrm{Z}} \\ \begin{pmatrix} 0.0742 \\ 1 \end{pmatrix} \end{array}$$

Matrix R

$$\begin{array}{c} \\ C_t \\ CapUt_t \\ I_t \\ L^{\mathrm{s}}_t \\ \Pi_t \\ U_t \\ W_t \\ Y_t \end{array} \begin{array}{c} \begin{array}{cc} K_{t-1} & Z_{t-1} \end{array} \\ \begin{pmatrix} 0.2823 & 0.4185 \\ -0.74 & 1.0041 \\ -1.1491 & 4.5972 \\ -0.2604 & 0.7601 \\ 0.9893 & 0.0146 \\ 0.0446 & 0.0408 \\ 0.1873 & 0.6958 \\ -0.0731 & 1.456 \end{pmatrix} \end{array}$$

Matrix S

$$\begin{array}{c} \\ C \\ CapUt \\ I \\ L^{\mathrm{s}} \\ \Pi \\ U \\ W \\ Y \end{array} \begin{array}{c} \epsilon^{\mathrm{Z}} \\ \begin{pmatrix} 0.4405 \\ 1.057 \\ 4.8392 \\ 0.8001 \\ 0.0153 \\ 0.0429 \\ 0.7325 \\ 1.5326 \end{pmatrix} \end{array}$$

10 Model statistics

10.1 Basic statistics

	Steady-state value	Std. dev.	Variance	Loglin
C	0.7449	0.0408	0.0017	Y
$CapUt$	0.9284	0.1001	0.01	Y
I	0.246	0.4485	0.2011	Y
K	10.96	0.0245	0.0006	Y
L^{s}	0.2673	0.0744	0.0055	Y
Π	11.0707	0.0242	0.0006	Y
U	-135.8123	0.004	0	Y
W	2.3722	0.0674	0.0045	Y
Y	0.9909	0.1414	0.02	Y
Z	1	0.0922	0.0085	Y

10.2 Correlation matrix

	C	$CapUt$	I	K	L^{s}	Π	U	W	Y	Z
C	1	0.939	0.973	0.387	0.967	0.172	0.995	0.995	0.983	0.986
$CapUt$		1	0.993	0.045	0.995	-0.178	0.898	0.969	0.986	0.984
I			1	0.164	1	-0.06	0.944	0.991	0.999	0.998
K				1	0.141	0.975	0.479	0.291	0.213	0.225
L^{s}					1	-0.083	0.936	0.988	0.997	0.996
Π						1	0.272	0.071	-0.01	0.003
U							1	0.979	0.96	0.963
W								1	0.997	0.998
Y									1	1
Z										1

10.3 Cross correlations with the reference variable (Y)

	$\sigma[\cdot]$ rel. to $\sigma[Y]$	Y_{t-5}	Y_{t-4}	Y_{t-3}	Y_{t-2}	Y_{t-1}	Y_t	Y_{t+1}	Y_{t+2}	Y_{t+3}	Y_{t+4}	Y_{t+5}
C_t	0.289	-0.117	0.01	0.178	0.393	0.66	0.983	0.748	0.543	0.366	0.217	0.095
$CapUt_t$	0.708	0.076	0.197	0.347	0.528	0.74	0.986	0.658	0.389	0.174	0.006	-0.12
I_t	3.172	0.01	0.135	0.294	0.489	0.723	0.999	0.699	0.448	0.242	0.079	-0.048
K_t	0.173	-0.54	-0.499	-0.413	-0.273	-0.068	0.213	0.408	0.531	0.595	0.613	0.596
L^{s}_t	0.526	0.023	0.148	0.305	0.497	0.727	0.997	0.692	0.437	0.229	0.064	-0.062
Π_t	0.171	-0.549	-0.535	-0.484	-0.386	-0.232	-0.01	0.255	0.436	0.548	0.603	0.614
U_t	0.029	-0.171	-0.046	0.124	0.344	0.621	0.96	0.758	0.575	0.414	0.275	0.156
W_t	0.477	-0.061	0.065	0.23	0.438	0.692	0.997	0.732	0.504	0.314	0.157	0.032
Y_t	1	-0.017	0.109	0.27	0.47	0.713	1	0.713	0.47	0.27	0.109	-0.017
Z_t	0.652	-0.024	0.102	0.264	0.465	0.71	1	0.716	0.476	0.277	0.116	-0.009

10.4 Autocorrelations

	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5
C	0.728	0.494	0.298	0.137	0.009
$CapUt$	0.714	0.472	0.272	0.111	-0.016
I	0.712	0.468	0.268	0.107	-0.019
K	0.96	0.864	0.731	0.576	0.414
L^{s}	0.711	0.468	0.268	0.106	-0.02
Π	0.964	0.869	0.737	0.583	0.419
U	0.743	0.518	0.326	0.165	0
W	0.717	0.478	0.279	0.118	-0.009
Y	0.713	0.47	0.27	0.109	-0.017
Z	0.713	0.471	0.271	0.11	-0.016

11 Impulse response functions

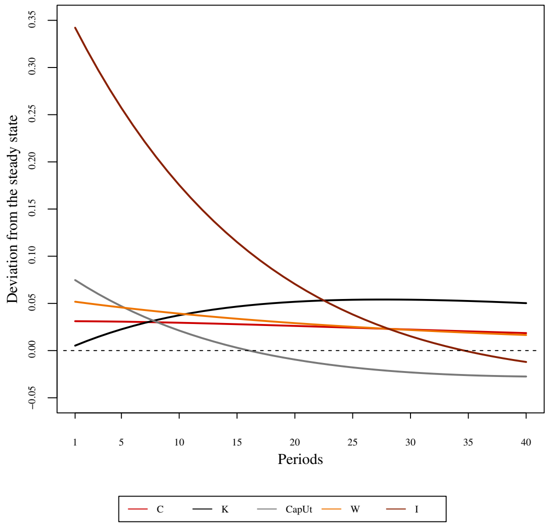

Figure 1: Impulse responses $(C, K, CapUt, W, I)$ to ϵ^{Z} shock

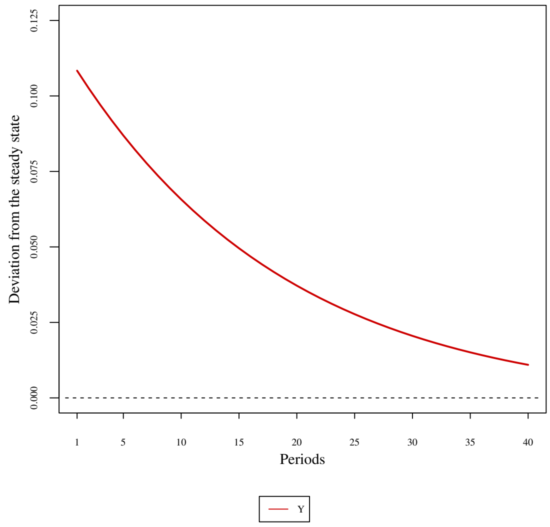

Figure 2: Impulse response (Y) to ϵ^{Z} shock