

1 CONSUMER

1.1 Optimisation problem

$$\max_{C_t, L_t^s} U_t = \beta \mathbf{E}_t [U_{t+1}] + (1 - \eta)^{-1} \left(C_t^\mu (1 - L_t^s)^{1-\mu} \right)^{1-\eta} \quad (1.1)$$

s.t. :

$$C_t = \pi_t + L_t^s W_t \quad (\lambda_t^c) \quad (1.2)$$

1.2 First order conditions

$$\beta - \lambda_t^U = 0 \quad (U_t) \quad (1.3)$$

$$-\lambda_t^c + \mu C_t^{-1+\mu} (1 - L_t^s)^{1-\mu} \left(C_t^\mu (1 - L_t^s)^{1-\mu} \right)^{-\eta} = 0 \quad (C_t) \quad (1.4)$$

$$\lambda_t^c W_t + (-1 + \mu) C_t^\mu (1 - L_t^s)^{-\mu} \left(C_t^\mu (1 - L_t^s)^{1-\mu} \right)^{-\eta} = 0 \quad (L_t^s) \quad (1.5)$$

2 FIRM

2.1 Optimisation problem

$$\max_{K_t, L_t^d, Y_t, I_t, \pi_t, CqU_t} \Pi_t = \pi_t + \lambda_t^{c-1} \mathbf{E}_t [\lambda_{t+1}^c \lambda_{t+1}^U \Pi_{t+1}] \quad (2.1)$$

s.t. :

$$Y_t = L_t^{d1-\alpha} Z_t^{1-\alpha} (K_{t-1} CqU_t)^\alpha \quad (\lambda_t^{\text{FIRM}^1}) \quad (2.2)$$

$$K_t = I_t + K_{t-1} (1 - \delta CqU_t)^\omega \quad (\lambda_t^{\text{FIRM}^2}) \quad (2.3)$$

$$\pi_t = -I_t - L_t^d W_t + P_t Y_t \quad (\lambda_t^{\text{FIRM}^3}) \quad (2.4)$$

2.2 First order conditions

$$-\lambda_t^{\text{FIRM}^\Pi} + \lambda_{t-1}^c \lambda_t^U = 0 \quad (\Pi_t) \quad (2.5)$$

$$-\lambda_t^{\text{FIRM}^2} + \text{E}_t \left[\lambda_{t+1}^{\text{FIRM}^\Pi} \left(\lambda_{t+1}^{\text{FIRM}^2} (1 - \delta \text{CqU}_{t+1}^\omega) + \alpha \lambda_{t+1}^{\text{FIRM}^1} \text{CqU}_{t+1} L_{t+1}^{\text{d}^{1-\alpha}} Z_{t+1}^{1-\alpha} (K_t \text{CqU}_{t+1})^{-1+\alpha} \right) \right] = 0 \quad (K_t) \quad (2.6)$$

$$-\lambda_t^{\text{FIRM}^3} W_t + \lambda_t^{\text{FIRM}^1} (1 - \alpha) L_t^{\text{d}^{-\alpha}} Z_t^{1-\alpha} (K_{t-1} \text{CqU}_t)^\alpha = 0 \quad (L_t^{\text{d}}) \quad (2.7)$$

$$-\lambda_t^{\text{FIRM}^1} + \lambda_t^{\text{FIRM}^3} P_t = 0 \quad (Y_t) \quad (2.8)$$

$$\lambda_t^{\text{FIRM}^2} - \lambda_t^{\text{FIRM}^3} = 0 \quad (I_t) \quad (2.9)$$

$$1 - \lambda_t^{\text{FIRM}^3} = 0 \quad (\pi_t) \quad (2.10)$$

$$-\delta \omega K_{t-1} \lambda_t^{\text{FIRM}^2} \text{CqU}_t^{-1+\omega} + \alpha K_{t-1} \lambda_t^{\text{FIRM}^1} L_t^{\text{d}^{1-\alpha}} Z_t^{1-\alpha} (K_{t-1} \text{CqU}_t)^{-1+\alpha} = 0 \quad (\text{CqU}_t) \quad (2.11)$$

2.3 First order conditions after reduction

$$-\lambda_t^{\text{FIRM}^\Pi} + \lambda_{t-1}^c \lambda_t^U = 0 \quad (\Pi_t) \quad (2.12)$$

$$-1 + \text{E}_t \left[\lambda_{t+1}^{\text{FIRM}^\Pi} \left(1 - \delta \text{CqU}_{t+1}^\omega + \alpha \lambda_{t+1}^{\text{FIRM}^1} \text{CqU}_{t+1} L_{t+1}^{\text{d}^{1-\alpha}} Z_{t+1}^{1-\alpha} (K_t \text{CqU}_{t+1})^{-1+\alpha} \right) \right] = 0 \quad (K_t) \quad (2.13)$$

$$-W_t + \lambda_t^{\text{FIRM}^1} (1 - \alpha) L_t^{\text{d}^{-\alpha}} Z_t^{1-\alpha} (K_{t-1} \text{CqU}_t)^\alpha = 0 \quad (L_t^{\text{d}}) \quad (2.14)$$

$$-\lambda_t^{\text{FIRM}^1} + P_t = 0 \quad (Y_t) \quad (2.15)$$

$$-\delta \omega K_{t-1} \text{CqU}_t^{-1+\omega} + \alpha K_{t-1} \lambda_t^{\text{FIRM}^1} L_t^{\text{d}^{1-\alpha}} Z_t^{1-\alpha} (K_{t-1} \text{CqU}_t)^{-1+\alpha} = 0 \quad (\text{CqU}_t) \quad (2.16)$$

3 EQUILIBRIUM

3.1 Identities

$$P_t = 1 \quad (3.1)$$

$$L_t^{\text{d}} = L_t^{\text{s}} \quad (3.2)$$

4 EXOG

4.1 Identities

$$Z_t = e^{\epsilon_t^Z + \phi \log Z_{t-1}} \quad (4.1)$$

5 Equilibrium relationships (after reduction)

$$-1 + \beta C_t^{1-\mu} (1 - L_t^s)^{-1+\mu} \left(C_t^\mu (1 - L_t^s)^{1-\mu} \right)^\eta \text{E}_t \left[\left(1 - \delta C_{qU} U_{t+1}^\omega + \alpha C_{qU} U_{t+1} L_{t+1}^{s1-\alpha} Z_{t+1}^{1-\alpha} (K_t C_{qU} U_{t+1})^{-1+\alpha} \right) C_{t+1}^{-1+\mu} (1 - L_{t+1}^s)^{1-\mu} \left(C_{t+1}^\mu (1 - L_{t+1}^s)^{1-\mu} \right)^{-\eta} \right] = 0 \quad (5.1)$$

$$-W_t + (1 - \alpha) L_t^{s1-\alpha} Z_t^{1-\alpha} (K_{t-1} C_{qU} U_t)^\alpha = 0 \quad (5.2)$$

$$-Y_t + L_t^{s1-\alpha} Z_t^{1-\alpha} (K_{t-1} C_{qU} U_t)^\alpha = 0 \quad (5.3)$$

$$-Z_t + e^{\epsilon_t^Z + \phi \log Z_{t-1}} = 0 \quad (5.4)$$

$$-\delta \omega K_{t-1} C_{qU} U_t^{-1+\omega} + \alpha K_{t-1} L_t^{s1-\alpha} Z_t^{1-\alpha} (K_{t-1} C_{qU} U_t)^{-1+\alpha} = 0 \quad (5.5)$$

$$(-1 + \mu) C_t^\mu (1 - L_t^s)^{-\mu} \left(C_t^\mu (1 - L_t^s)^{1-\mu} \right)^{-\eta} + \mu W_t C_t^{-1+\mu} (1 - L_t^s)^{1-\mu} \left(C_t^\mu (1 - L_t^s)^{1-\mu} \right)^{-\eta} = 0 \quad (5.6)$$

$$I_t - K_t + K_{t-1} (1 - \delta C_{qU} U_t^\omega) = 0 \quad (5.7)$$

$$U_t - \beta \text{E}_t [U_{t+1}] - (1 - \eta)^{-1} \left(C_t^\mu (1 - L_t^s)^{1-\mu} \right)^{1-\eta} = 0 \quad (5.8)$$

$$-C_t + \Pi_t + L_t^s W_t - \beta (C_t^{-1+\mu})^{-1} \left((1 - L_t^s)^{1-\mu} \right)^{-1} \left(\left(C_t^\mu (1 - L_t^s)^{1-\mu} \right)^{-\eta} \right)^{-1} \text{E}_t \left[\Pi_{t+1} C_{t+1}^{-1+\mu} (1 - L_{t+1}^s)^{1-\mu} \left(C_{t+1}^\mu (1 - L_{t+1}^s)^{1-\mu} \right)^{-\eta} \right] = 0 \quad (5.9)$$

$$-I_t - \Pi_t + Y_t - L_t^s W_t + \beta (C_t^{-1+\mu})^{-1} \left((1 - L_t^s)^{1-\mu} \right)^{-1} \left(\left(C_t^\mu (1 - L_t^s)^{1-\mu} \right)^{-\eta} \right)^{-1} \text{E}_t \left[\Pi_{t+1} C_{t+1}^{-1+\mu} (1 - L_{t+1}^s)^{1-\mu} \left(C_{t+1}^\mu (1 - L_{t+1}^s)^{1-\mu} \right)^{-\eta} \right] = 0 \quad (5.10)$$

6 Steady state relationships (after reduction)

$$-1 + \beta \left(1 - \delta C_{\text{ap}} U_{\text{ss}}^{\omega} + \alpha C_{\text{ap}} U_{\text{ss}} L_{\text{ss}}^{s\ 1-\alpha} Z_{\text{ss}}^{1-\alpha} (C_{\text{ap}} U_{\text{ss}} K_{\text{ss}})^{-1+\alpha} \right) C_{\text{ss}}^{-1+\mu} C_{\text{ss}}^{1-\mu} (1 - L_{\text{ss}}^s)^{-1+\mu} (1 - L_{\text{ss}}^s)^{1-\mu} = 0 \quad (6.1)$$

$$-W_{\text{ss}} + (1 - \alpha) L_{\text{ss}}^s{}^{-\alpha} Z_{\text{ss}}^{1-\alpha} (C_{\text{ap}} U_{\text{ss}} K_{\text{ss}})^{\alpha} = 0 \quad (6.2)$$

$$-Y_{\text{ss}} + L_{\text{ss}}^s{}^{1-\alpha} Z_{\text{ss}}^{1-\alpha} (C_{\text{ap}} U_{\text{ss}} K_{\text{ss}})^{\alpha} = 0 \quad (6.3)$$

$$-Z_{\text{ss}} + e^{\phi \log Z_{\text{ss}}} = 0 \quad (6.4)$$

$$-\delta \omega K_{\text{ss}} C_{\text{ap}} U_{\text{ss}}^{-1+\omega} + \alpha K_{\text{ss}} L_{\text{ss}}^s{}^{1-\alpha} Z_{\text{ss}}^{1-\alpha} (C_{\text{ap}} U_{\text{ss}} K_{\text{ss}})^{-1+\alpha} = 0 \quad (6.5)$$

$$(-1 + \mu) C_{\text{ss}}^{\mu} (1 - L_{\text{ss}}^s)^{-\mu} \left(C_{\text{ss}}^{\mu} (1 - L_{\text{ss}}^s)^{1-\mu} \right)^{-\eta} + \mu W_{\text{ss}} C_{\text{ss}}^{-1+\mu} (1 - L_{\text{ss}}^s)^{1-\mu} \left(C_{\text{ss}}^{\mu} (1 - L_{\text{ss}}^s)^{1-\mu} \right)^{-\eta} = 0 \quad (6.6)$$

$$I_{\text{ss}} - K_{\text{ss}} + K_{\text{ss}} (1 - \delta C_{\text{ap}} U_{\text{ss}}^{\omega}) = 0 \quad (6.7)$$

$$U_{\text{ss}} - \beta U_{\text{ss}} - (1 - \eta)^{-1} \left(C_{\text{ss}}^{\mu} (1 - L_{\text{ss}}^s)^{1-\mu} \right)^{1-\eta} = 0 \quad (6.8)$$

$$-C_{\text{ss}} + \Pi_{\text{ss}} + L_{\text{ss}}^s W_{\text{ss}} - \beta \Pi_{\text{ss}} (1 - L_{\text{ss}}^s)^{-1+\mu} (1 - L_{\text{ss}}^s)^{1-\mu} = 0 \quad (6.9)$$

$$-I_{\text{ss}} - \Pi_{\text{ss}} + Y_{\text{ss}} - L_{\text{ss}}^s W_{\text{ss}} + \beta \Pi_{\text{ss}} C_{\text{ss}}^{-1+\mu} C_{\text{ss}}^{1-\mu} (1 - L_{\text{ss}}^s)^{-1+\mu} (1 - L_{\text{ss}}^s)^{1-\mu} = 0 \quad (6.10)$$

7 Parameter settings

$$\alpha = 0.36 \quad (7.1)$$

$$\beta = 0.99 \quad (7.2)$$

$$\delta = 0.025 \quad (7.3)$$

$$\eta = 2 \quad (7.4)$$

$$\mu = 0.3 \quad (7.5)$$

$$\omega = 1.45 \tag{7.6}$$

$$\phi = 0.95 \tag{7.7}$$

8 Steady-state values

	Steady-state value
C	0.7449
$CqUt$	0.9284
I	0.246
K	10.96
L^s	0.2673
Π	11.0707
U	-135.8123
W	2.3722
Y	0.9909
Z	1

9 The solution of the 1st order perturbation

Matrix P

$$\begin{matrix} & K_{t-1} & Z_{t-1} \\ K_t & \left(\begin{array}{cc} 0.9758 & 0.0705 \\ 0 & 0.95 \end{array} \right) \\ Z_t & & \end{matrix}$$

Matrix Q

$$\begin{matrix} & \epsilon^Z \\ K & \left(\begin{array}{c} 0.0742 \\ 1 \end{array} \right) \\ Z & \end{matrix}$$

Matrix R

$$\begin{matrix} & K_{t-1} & Z_{t-1} \\ C_t & \left(\begin{array}{cc} 0.2823 & 0.4185 \\ -0.74 & 1.0041 \\ -1.1491 & 4.5972 \\ -0.2604 & 0.7601 \\ 0.9893 & 0.0146 \\ 0.0446 & 0.0408 \\ 0.1873 & 0.6958 \\ -0.0731 & 1.456 \end{array} \right) \\ CqUt_t & & \\ I_t & & \\ L_t^s & & \\ \Pi_t & & \\ U_t & & \\ W_t & & \\ Y_t & & \end{matrix}$$

Matrix S

$$\begin{matrix} & \epsilon^Z \\ C & \left(\begin{array}{c} 0.4405 \\ 1.057 \\ 4.8392 \\ 0.8001 \\ 0.0153 \\ 0.0429 \\ 0.7325 \\ 1.5326 \end{array} \right) \\ CqUt & & \\ I & & \\ L^s & & \\ \Pi & & \\ U & & \\ W & & \\ Y & & \end{matrix}$$

10 Model statistics

10.1 Basic statistics

	Steady-state value	Std. dev.	Variance	Loglin
C	0.7449	0.0408	0.0017	Y
$CqUt$	0.9284	0.1001	0.01	Y
I	0.246	0.4485	0.2011	Y
K	10.96	0.0245	0.0006	Y
L^s	0.2673	0.0744	0.0055	Y
Π	11.0707	0.0242	0.0006	Y
U	-135.8123	0.004	0	Y
W	2.3722	0.0674	0.0045	Y
Y	0.9909	0.1414	0.02	Y
Z	1	0.0922	0.0085	Y

10.2 Correlation matrix

	C	$CqUt$	I	K	L^s	Π	U	W	Y	Z
C	1	0.939	0.973	0.387	0.967	0.172	0.995	0.995	0.983	0.986
$CqUt$		1	0.993	0.045	0.995	-0.178	0.898	0.969	0.986	0.984
I			1	0.164	1	-0.06	0.944	0.991	0.999	0.998
K				1	0.141	0.975	0.479	0.291	0.213	0.225
L^s					1	-0.083	0.936	0.988	0.997	0.996
Π						1	0.272	0.071	-0.01	0.003
U							1	0.979	0.96	0.963
W								1	0.997	0.998
Y									1	1
Z										1

10.3 Cross correlations with the reference variable (Y)

	$\sigma[\cdot]$ rel. to $\sigma[Y]$	Y_{t-5}	Y_{t-4}	Y_{t-3}	Y_{t-2}	Y_{t-1}	Y_t	Y_{t+1}	Y_{t+2}	Y_{t+3}	Y_{t+4}	Y_{t+5}
C_t	0.289	-0.117	0.01	0.178	0.393	0.66	0.983	0.748	0.543	0.366	0.217	0.095
$CqUt_t$	0.708	0.076	0.197	0.347	0.528	0.74	0.986	0.658	0.389	0.174	0.006	-0.12
I_t	3.172	0.01	0.135	0.294	0.489	0.723	0.999	0.699	0.448	0.242	0.079	-0.048
K_t	0.173	-0.54	-0.499	-0.413	-0.273	-0.068	0.213	0.408	0.531	0.595	0.613	0.596
L_t^s	0.526	0.023	0.148	0.305	0.497	0.727	0.997	0.692	0.437	0.229	0.064	-0.062
Π_t	0.171	-0.549	-0.535	-0.484	-0.386	-0.232	-0.01	0.255	0.436	0.548	0.603	0.614
U_t	0.029	-0.171	-0.046	0.124	0.344	0.621	0.96	0.758	0.575	0.414	0.275	0.156
W_t	0.477	-0.061	0.065	0.23	0.438	0.692	0.997	0.732	0.504	0.314	0.157	0.032
Y_t	1	-0.017	0.109	0.27	0.47	0.713	1	0.713	0.47	0.27	0.109	-0.017
Z_t	0.652	-0.024	0.102	0.264	0.465	0.71	1	0.716	0.476	0.277	0.116	-0.009

10.4 Autocorrelations

	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5
C	0.728	0.494	0.298	0.137	0.009
$CqUt$	0.714	0.472	0.272	0.111	-0.016
I	0.712	0.468	0.268	0.107	-0.019
K	0.96	0.864	0.731	0.576	0.414
L^s	0.711	0.468	0.268	0.106	-0.02
Π	0.964	0.869	0.737	0.583	0.419
U	0.743	0.518	0.326	0.165	0
W	0.717	0.478	0.279	0.118	-0.009
Y	0.713	0.47	0.27	0.109	-0.017
Z	0.713	0.471	0.271	0.11	-0.016

11 Impulse response functions

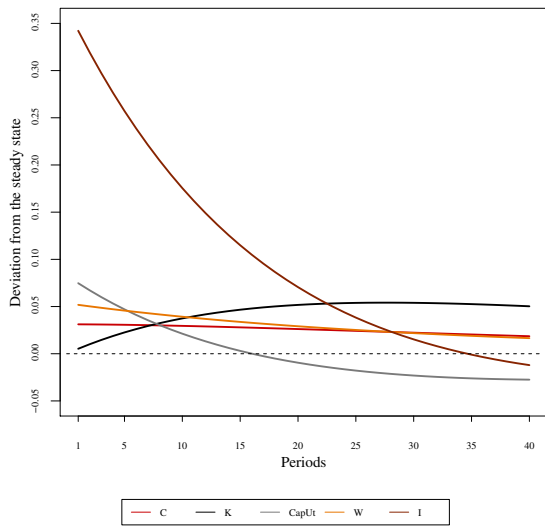


Figure 1: Impulse responses ($C, K, CapUt, W, I$) to ϵ^Z shock

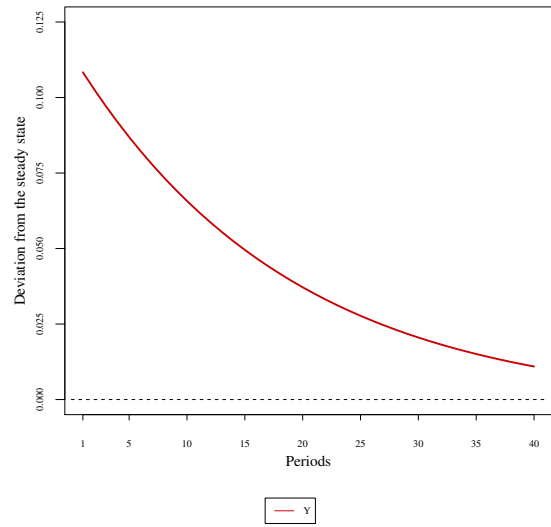


Figure 2: Impulse response (Y) to ϵ^Z shock