

1 CONSUMER

1.1 Optimisation problem

$$\max_{K_t^s, C_t, L_t^s, I_t, H_t} U_t = \beta \mathbf{E}_t [U_{t+1}] + (1 - \eta)^{-1} \left((1 - L_t^s)^{1-\mu} (C_t - \text{pers}H_t)^\mu \right)^{1-\eta} \quad (1.1)$$

s.t. :

$$C_t + I_t = \pi_t + K_{t-1}^s r_t + L_t^s W_t \quad \left(\lambda_t^{\text{CONSUMER}^1} \right) \quad (1.2)$$

$$K_t^s = I_t + K_{t-1}^s (1 - \delta) \quad \left(\lambda_t^{\text{CONSUMER}^2} \right) \quad (1.3)$$

$$H_t = C_{t-1} \quad \left(\lambda_t^{\text{CONSUMER}^3} \right) \quad (1.4)$$

1.2 First order conditions

$$-\lambda_t^{\text{CONSUMER}^2} + \beta \left((1 - \delta) \mathbf{E}_t \left[\lambda_{t+1}^{\text{CONSUMER}^2} \right] + \mathbf{E}_t \left[\lambda_{t+1}^{\text{CONSUMER}^1} r_{t+1} \right] \right) = 0 \quad (K_t^s) \quad (1.5)$$

$$-\lambda_t^{\text{CONSUMER}^1} + \beta \mathbf{E}_t \left[\lambda_{t+1}^{\text{CONSUMER}^3} \right] + \mu (1 - L_t^s)^{1-\mu} (C_t - \text{pers}H_t)^{-1+\mu} \left((1 - L_t^s)^{1-\mu} (C_t - \text{pers}H_t)^\mu \right)^{-\eta} = 0 \quad (C_t) \quad (1.6)$$

$$\lambda_t^{\text{CONSUMER}^1} W_t + (-1 + \mu) (1 - L_t^s)^{-\mu} (C_t - \text{pers}H_t)^\mu \left((1 - L_t^s)^{1-\mu} (C_t - \text{pers}H_t)^\mu \right)^{-\eta} = 0 \quad (L_t^s) \quad (1.7)$$

$$-\lambda_t^{\text{CONSUMER}^1} + \lambda_t^{\text{CONSUMER}^2} = 0 \quad (I_t) \quad (1.8)$$

$$-\lambda_t^{\text{CONSUMER}^3} - \mu \text{pers} (1 - L_t^s)^{1-\mu} (C_t - \text{pers}H_t)^{-1+\mu} \left((1 - L_t^s)^{1-\mu} (C_t - \text{pers}H_t)^\mu \right)^{-\eta} = 0 \quad (H_t) \quad (1.9)$$

2 FIRM

2.1 Optimisation problem

$$\max_{K_t^d, L_t^d, Y_t} \pi_t = Y_t - L_t^d W_t - r_t K_t^d \quad (2.1)$$

s.t. :

$$Y_t = Z_t K_t^{\text{d}\alpha} L_t^{\text{d}1-\alpha} \quad \left(\lambda_t^{\text{FIRM}^1} \right) \quad (2.2)$$

2.2 First order conditions

$$-r_t + \alpha \lambda_t^{\text{FIRM}^1} Z_t K_t^{\text{d}-1+\alpha} L_t^{\text{d}1-\alpha} = 0 \quad (K_t^{\text{d}}) \quad (2.3)$$

$$-W_t + \lambda_t^{\text{FIRM}^1} Z_t (1 - \alpha) K_t^{\text{d}\alpha} L_t^{\text{d}-\alpha} = 0 \quad (L_t^{\text{d}}) \quad (2.4)$$

$$1 - \lambda_t^{\text{FIRM}^1} = 0 \quad (Y_t) \quad (2.5)$$

2.3 First order conditions after reduction

$$-r_t + \alpha Z_t K_t^{\text{d}-1+\alpha} L_t^{\text{d}1-\alpha} = 0 \quad (K_t^{\text{d}}) \quad (2.6)$$

$$-W_t + Z_t (1 - \alpha) K_t^{\text{d}\alpha} L_t^{\text{d}-\alpha} = 0 \quad (L_t^{\text{d}}) \quad (2.7)$$

3 EQUILIBRIUM

3.1 Identities

$$K_t^{\text{d}} = K_{t-1}^{\text{s}} \quad (3.1)$$

$$L_t^{\text{d}} = L_t^{\text{s}} \quad (3.2)$$

4 EXOG

4.1 Identities

$$Z_t = e^{\epsilon_t^Z + \phi \log Z_{t-1}} \quad (4.1)$$

5 Equilibrium relationships (after reduction)

$$C_{t-1} - H_t = 0 \quad (5.1)$$

$$-\lambda_t^{\text{CONSUMER}^2} + \beta \left((1 - \delta) \mathbb{E}_t \left[\lambda_{t+1}^{\text{CONSUMER}^2} \right] + \mathbb{E}_t \left[\lambda_{t+1}^{\text{CONSUMER}^2} r_{t+1} \right] \right) = 0 \quad (5.2)$$

$$-r_t + \alpha Z_t K_{t-1}^{\text{s}-1+\alpha} L_t^{\text{s}1-\alpha} = 0 \quad (5.3)$$

$$-W_t + Z_t (1 - \alpha) K_{t-1}^{\text{s}\alpha} L_t^{\text{s}-\alpha} = 0 \quad (5.4)$$

$$-Y_t + Z_t K_{t-1}^s \alpha L_t^{s1-\alpha} = 0 \quad (5.5)$$

$$-Z_t + e^{\epsilon_t^Z + \phi \log Z_{t-1}} = 0 \quad (5.6)$$

$$\lambda_t^{\text{CONSUMER}^2} W_t + (-1 + \mu) (1 - L_t^s)^{-\mu} (C_t - \text{pers} H_t)^\mu \left((1 - L_t^s)^{1-\mu} (C_t - \text{pers} H_t)^\mu \right)^{-\eta} = 0 \quad (5.7)$$

$$-\lambda_t^{\text{CONSUMER}^2} - \beta \mu \text{pers} E_t \left[(1 - L_{t+1}^s)^{1-\mu} (C_{t+1} - \text{pers} H_{t+1})^{-1+\mu} \left((1 - L_{t+1}^s)^{1-\mu} (C_{t+1} - \text{pers} H_{t+1})^\mu \right)^{-\eta} \right] + \mu (1 - L_t^s)^{1-\mu} (C_t - \text{pers} H_t)^{-1+\mu} \left((1 - L_t^s)^{1-\mu} (C_t - \text{pers} H_t)^\mu \right)^{-\eta} = 0 \quad (5.8)$$

$$-C_t - I_t + Y_t = 0 \quad (5.9)$$

$$I_t - K_t^s + K_{t-1}^s (1 - \delta) = 0 \quad (5.10)$$

$$U_t - \beta E_t [U_{t+1}] - (1 - \eta)^{-1} \left((1 - L_t^s)^{1-\mu} (C_t - \text{pers} H_t)^\mu \right)^{1-\eta} = 0 \quad (5.11)$$

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6 Steady state relationships (after reduction)

$$C_{ss} - H_{ss} = 0 \quad (6.1)$$

$$-\lambda_{ss}^{\text{CONSUMER}^2} + \beta \left(\lambda_{ss}^{\text{CONSUMER}^2} r_{ss} + \lambda_{ss}^{\text{CONSUMER}^2} (1 - \delta) \right) = 0 \quad (6.2)$$

$$-r_{ss} + \alpha Z_{ss} K_{ss}^{s-1+\alpha} L_{ss}^{s1-\alpha} = 0 \quad (6.3)$$

$$-W_{ss} + Z_{ss} (1 - \alpha) K_{ss}^{s\alpha} L_{ss}^{s-\alpha} = 0 \quad (6.4)$$

$$-Y_{ss} + Z_{ss} K_{ss}^{s\alpha} L_{ss}^{s1-\alpha} = 0 \quad (6.5)$$

$$-Z_{ss} + e^{\phi \log Z_{ss}} = 0 \quad (6.6)$$

$$\lambda_{ss}^{\text{CONSUMER}^2} W_{ss} + (-1 + \mu) (1 - L_{ss}^s)^{-\mu} (C_{ss} - \text{pers} H_{ss})^\mu \left((1 - L_{ss}^s)^{1-\mu} (C_{ss} - \text{pers} H_{ss})^\mu \right)^{-\eta} = 0 \quad (6.7)$$

$$-\lambda_{ss}^{\text{CONSUMER}^2} + \mu(1 - L_{ss}^s)^{1-\mu}(C_{ss} - persH_{ss})^{-1+\mu} \left((1 - L_{ss}^s)^{1-\mu}(C_{ss} - persH_{ss})^\mu \right)^{-\eta} - \beta \mu pers(1 - L_{ss}^s)^{1-\mu}(C_{ss} - persH_{ss})^{-1+\mu} \left((1 - L_{ss}^s)^{1-\mu}(C_{ss} - persH_{ss})^\mu \right)^{-\eta} = 0 \quad (6.8)$$

$$-C_{ss} - I_{ss} + Y_{ss} = 0 \quad (6.9)$$

$$I_{ss} - K_{ss}^s + K_{ss}^s(1 - \delta) = 0 \quad (6.10)$$

$$U_{ss} - \beta U_{ss} - (1 - \eta)^{-1} \left((1 - L_{ss}^s)^{1-\mu}(C_{ss} - persH_{ss})^\mu \right)^{1-\eta} = 0 \quad (6.11)$$

7 Calibrating equations

$$-0.36Y_{ss} + r_{ss}K_{ss}^s = 0 \quad (7.1)$$

8 Parameter settings

$$\beta = 0.99 \quad (8.1)$$

$$\delta = 0.025 \quad (8.2)$$

$$\eta = 2 \quad (8.3)$$

$$\mu = 0.3 \quad (8.4)$$

$$pers = 0.57 \quad (8.5)$$

$$\phi = 0.95 \quad (8.6)$$

9 Steady-state values

	Steady-state value
$\lambda^{\text{CONSUMER}^2}$	0.7116
r	0.0351
C	0.7494
H	0.7494
I	0.2584
K^s	10.3356
L^s	0.2721
U	-175.4236
W	2.3706
Y	1.0078
Z	1

10 The solution of the 1st order perturbation

Matrix P

$$\begin{matrix} C_t \\ K_t^s \\ Z_t \end{matrix} \begin{pmatrix} C_{t-1} & K_{t-1}^s & Z_{t-1} \\ 0.5544 & 0.0151 & 0.1764 \\ -0.5092 & 0.9817 & 1.1759 \\ 0 & 0 & 0.95 \end{pmatrix}$$

Matrix Q

$$\begin{matrix} C \\ K^s \\ Z \end{matrix} \begin{pmatrix} 0.1857 \\ 1.2377 \\ 1 \end{pmatrix} \epsilon^Z$$

Matrix R

$$\begin{matrix} \lambda_t^{\text{CONSUMER}^2} \\ r_t \\ H_t \\ I_t \\ L_t^s \\ U_t \\ W_t \\ Y_t \end{matrix} \begin{pmatrix} C_{t-1} & K_{t-1}^s & Z_{t-1} \\ 0.0599 & -0.0494 & -0.3592 \\ 0.0016 & -0.0026 & 0.0471 \\ 1 & 0 & 0 \\ -0.5092 & 0.0067 & 1.1759 \\ 0.0191 & -0.0056 & 0.1666 \\ -0.9309 & 0.7188 & 11.4498 \\ -0.0598 & 0.1002 & 1.7296 \\ 0.0452 & 0.0218 & 1.3522 \end{pmatrix}$$

Matrix S

$$\begin{matrix} \lambda^{\text{CONSUMER}^2} \\ r \\ H \\ I \\ L^s \\ U \\ W \\ Y \end{matrix} \begin{pmatrix} -0.3781 \\ 0.0496 \\ 0 \\ 1.2377 \\ 0.1753 \\ 12.0524 \\ 1.8206 \\ 1.4234 \end{pmatrix} \epsilon^Z$$

11 Model statistics

11.1 Basic statistics

	Steady-state value	Std. dev.	Variance	Loglin
r	0.0351	0.0046	0	N
C	0.7494	0.0333	0.0011	N
H	0.7494	0.0333	0.0011	N
I	0.2584	0.1077	0.0116	N
K^s	10.3356	0.3633	0.132	N
L^s	0.2721	0.0164	0.0003	N
U	-175.4236	1.1325	1.2825	N
W	2.3706	0.1719	0.0295	N
Y	1.0078	0.1325	0.0175	N
Z	1	0.0922	0.0085	N

11.2 Correlation matrix

	r	C	H	I	K^s	L^s	U	W	Y	Z
r	1	0.623	0.263	0.993	0.14	0.994	0.918	0.92	0.965	0.98
C		1	0.911	0.674	0.826	0.701	0.867	0.865	0.799	0.758
H			1	0.315	0.899	0.358	0.584	0.582	0.485	0.429
I				1	0.232	0.997	0.95	0.951	0.983	0.992
K^s					1	0.243	0.521	0.516	0.396	0.332
L^s						1	0.954	0.956	0.987	0.996
U							1	1	0.99	0.978
W								1	0.991	0.979
Y									1	0.998
Z										1

11.3 Cross correlations with the reference variable (Y)

	$\sigma[\cdot]$ rel. to $\sigma[Y]$	Y_{t-5}	Y_{t-4}	Y_{t-3}	Y_{t-2}	Y_{t-1}	Y_t	Y_{t+1}	Y_{t+2}	Y_{t+3}	Y_{t+4}	Y_{t+5}
r_t	0.035	0.126	0.245	0.388	0.556	0.749	0.965	0.613	0.331	0.109	-0.059	-0.182
C_t	0.251	-0.244	-0.131	0.025	0.228	0.485	0.799	0.867	0.804	0.677	0.527	0.375
H_t	0.251	-0.32	-0.244	-0.131	0.025	0.228	0.485	0.799	0.867	0.804	0.677	0.527
I_t	0.813	0.069	0.193	0.345	0.527	0.74	0.983	0.622	0.349	0.143	-0.011	-0.122
K_t^s	2.742	-0.459	-0.39	-0.278	-0.115	0.107	0.396	0.57	0.66	0.685	0.665	0.612
L_t^s	0.124	0.076	0.2	0.352	0.534	0.746	0.987	0.668	0.402	0.185	0.016	-0.113
U_t	8.55	-0.077	0.053	0.222	0.432	0.688	0.99	0.754	0.545	0.366	0.214	0.089
W_t	1.298	-0.073	0.057	0.226	0.436	0.69	0.991	0.755	0.546	0.365	0.211	0.085
Y_t	1	-0.005	0.124	0.286	0.486	0.724	1	0.724	0.486	0.286	0.124	-0.005
Z_t	0.696	0.028	0.155	0.314	0.507	0.735	0.998	0.698	0.446	0.24	0.076	-0.051

11.4 Autocorrelations

	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5
r	0.707	0.462	0.261	0.101	0
C	0.911	0.738	0.537	0.337	0.156
H	0.911	0.738	0.537	0.337	0.156
I	0.667	0.413	0.217	0.069	-0.043
K^s	0.955	0.851	0.71	0.55	0.384
L^s	0.716	0.473	0.271	0.108	-0.02
U	0.737	0.509	0.316	0.155	0.025
W	0.74	0.512	0.317	0.155	0.024
Y	0.724	0.486	0.286	0.124	-0.005
Z	0.713	0.471	0.271	0.11	-0.016

12 Impulse response functions

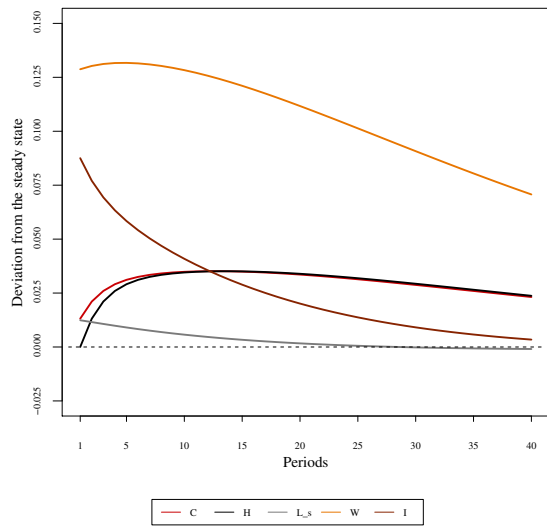


Figure 1: Impulse responses (C, H, L^s, W, I) to ϵ^Z shock

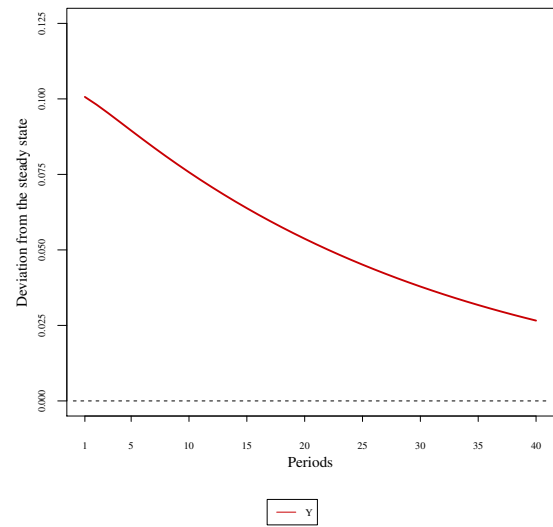


Figure 2: Impulse response (Y) to ϵ^Z shock