

1 CONSUMER

1.1 Optimisation problem

$$\max_{K_t^s, C_t, L_t^s, I_t, H_t} U_t = \beta \mathbf{E}_t [U_{t+1}] + (1 - \eta)^{-1} \left((1 - L_t^s)^{1-\mu} (C_t - \text{pers}H_t)^\mu \right)^{1-\eta} \quad (1.1)$$

s.t. :

$$C_t + I_t = \pi_t + K_{t-1}^s r_t + L_t^s W_t \quad \left(\lambda_t^{\text{CONSUMER}^1} \right) \quad (1.2)$$

$$K_t^s = I_t + K_{t-1}^s (1 - \delta) \quad \left(\lambda_t^{\text{CONSUMER}^2} \right) \quad (1.3)$$

$$H_t = C_{t-1} \quad \left(\lambda_t^{\text{CONSUMER}^3} \right) \quad (1.4)$$

1.2 First order conditions

$$-\lambda_t^{\text{CONSUMER}^2} + \beta \left((1 - \delta) \mathbf{E}_t \left[\lambda_{t+1}^{\text{CONSUMER}^2} \right] + \mathbf{E}_t \left[\lambda_{t+1}^{\text{CONSUMER}^1} r_{t+1} \right] \right) = 0 \quad (K_t^s) \quad (1.5)$$

$$-\lambda_t^{\text{CONSUMER}^1} + \beta \mathbf{E}_t \left[\lambda_{t+1}^{\text{CONSUMER}^3} \right] + \mu (1 - L_t^s)^{1-\mu} (C_t - \text{pers}H_t)^{-1+\mu} \left((1 - L_t^s)^{1-\mu} (C_t - \text{pers}H_t)^\mu \right)^{-\eta} = 0 \quad (C_t) \quad (1.6)$$

$$\lambda_t^{\text{CONSUMER}^1} W_t + (-1 + \mu) (1 - L_t^s)^{-\mu} (C_t - \text{pers}H_t)^\mu \left((1 - L_t^s)^{1-\mu} (C_t - \text{pers}H_t)^\mu \right)^{-\eta} = 0 \quad (L_t^s) \quad (1.7)$$

$$-\lambda_t^{\text{CONSUMER}^1} + \lambda_t^{\text{CONSUMER}^2} = 0 \quad (I_t) \quad (1.8)$$

$$-\lambda_t^{\text{CONSUMER}^3} - \mu \text{pers} (1 - L_t^s)^{1-\mu} (C_t - \text{pers}H_t)^{-1+\mu} \left((1 - L_t^s)^{1-\mu} (C_t - \text{pers}H_t)^\mu \right)^{-\eta} = 0 \quad (H_t) \quad (1.9)$$

2 FIRM

2.1 Optimisation problem

$$\max_{K_t^d, L_t^d, Y_t} \pi_t = Y_t - L_t^d W_t - r_t K_t^d \quad (2.1)$$

s.t. :

$$Y_t = Z_t K_t^{\text{d}\alpha} L_t^{\text{d}1-\alpha} \quad \left(\lambda_t^{\text{FIRM}^1} \right) \quad (2.2)$$

2.2 First order conditions

$$-r_t + \alpha \lambda_t^{\text{FIRM}^1} Z_t K_t^{\text{d}^{-1+\alpha}} L_t^{\text{d}^{1-\alpha}} = 0 \quad (K_t^{\text{d}}) \quad (2.3)$$

$$-W_t + \lambda_t^{\text{FIRM}^1} Z_t (1 - \alpha) K_t^{\text{d}^\alpha} L_t^{\text{d}^{-\alpha}} = 0 \quad (L_t^{\text{d}}) \quad (2.4)$$

$$1 - \lambda_t^{\text{FIRM}^1} = 0 \quad (Y_t) \quad (2.5)$$

2.3 First order conditions after reduction

$$-r_t + \alpha Z_t K_t^{\text{d}^{-1+\alpha}} L_t^{\text{d}^{1-\alpha}} = 0 \quad (K_t^{\text{d}}) \quad (2.6)$$

$$-W_t + Z_t (1 - \alpha) K_t^{\text{d}^\alpha} L_t^{\text{d}^{-\alpha}} = 0 \quad (L_t^{\text{d}}) \quad (2.7)$$

3 EQUILIBRIUM

3.1 Identities

$$K_t^{\text{d}} = K_{t-1}^{\text{s}} \quad (3.1)$$

$$L_t^{\text{d}} = L_t^{\text{s}} \quad (3.2)$$

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4 EXOG

4.1 Identities

$$Z_t = e^{\epsilon_t^Z + \phi \log Z_{t-1}} \quad (4.1)$$

5 Equilibrium relationships (after reduction)

$$C_{t-1} - H_t = 0 \quad (5.1)$$

$$-\lambda_t^{\text{CONSUMER}^2} + \beta \left((1 - \delta) \text{E}_t \left[\lambda_{t+1}^{\text{CONSUMER}^2} \right] + \text{E}_t \left[\lambda_{t+1}^{\text{CONSUMER}^2} r_{t+1} \right] \right) = 0 \quad (5.2)$$

$$-r_t + \alpha Z_t K_{t-1}^{\text{s}^{-1+\alpha}} L_t^{\text{s}^{1-\alpha}} = 0 \quad (5.3)$$

$$-W_t + Z_t (1 - \alpha) K_{t-1}^{\text{s}^\alpha} L_t^{\text{s}^{-\alpha}} = 0 \quad (5.4)$$

$$-Y_t + Z_t K_{t-1}^{\text{s}^\alpha} L_t^{\text{s}^{1-\alpha}} = 0 \quad (5.5)$$

$$-Z_t + e^{\epsilon_t^Z + \phi \log Z_{t-1}} = 0 \quad (5.6)$$

$$\lambda_t^{\text{CONSUMER}^2} W_t + (-1 + \mu) (1 - L_t^s)^{-\mu} (C_t - \text{pers} H_t)^\mu \left((1 - L_t^s)^{1-\mu} (C_t - \text{pers} H_t)^\mu \right)^{-\eta} = 0 \quad (5.7)$$

$$-\lambda_t^{\text{CONSUMER}^2} - \beta \mu \text{pers} E_t \left[(1 - L_{t+1}^s)^{1-\mu} (C_{t+1} - \text{pers} H_{t+1})^{-1+\mu} \left((1 - L_{t+1}^s)^{1-\mu} (C_{t+1} - \text{pers} H_{t+1})^\mu \right)^{-\eta} \right] + \mu (1 - L_t^s)^{1-\mu} (C_t - \text{pers} H_t)^{-1+\mu} \left((1 - L_t^s)^{1-\mu} (C_t - \text{pers} H_t)^\mu \right)^{-\eta} = 0 \quad (5.8)$$

$$-C_t - I_t + Y_t = 0 \quad (5.9)$$

$$I_t - K_t^s + K_{t-1}^s (1 - \delta) = 0 \quad (5.10)$$

$$U_t - \beta E_t [U_{t+1}] - (1 - \eta)^{-1} \left((1 - L_t^s)^{1-\mu} (C_t - \text{pers} H_t)^\mu \right)^{1-\eta} = 0 \quad (5.11)$$

6 Steady state relationships (after reduction)

$$C_{ss} - H_{ss} = 0 \quad (6.1)$$

$$-\lambda_{ss}^{\text{CONSUMER}^2} + \beta \left(\lambda_{ss}^{\text{CONSUMER}^2} r_{ss} + \lambda_{ss}^{\text{CONSUMER}^2} (1 - \delta) \right) = 0 \quad (6.2)$$

$$-r_{ss} + \alpha Z_{ss} K_{ss}^s^{-1+\alpha} L_{ss}^s^{1-\alpha} = 0 \quad (6.3)$$

$$-W_{ss} + Z_{ss} (1 - \alpha) K_{ss}^s^\alpha L_{ss}^s^{-\alpha} = 0 \quad (6.4)$$

$$-Y_{ss} + Z_{ss} K_{ss}^s^\alpha L_{ss}^s^{1-\alpha} = 0 \quad (6.5)$$

$$-Z_{ss} + e^{\phi \log Z_{ss}} = 0 \quad (6.6)$$

$$\lambda_{ss}^{\text{CONSUMER}^2} W_{ss} + (-1 + \mu) (1 - L_{ss}^s)^{-\mu} (C_{ss} - \text{pers} H_{ss})^\mu \left((1 - L_{ss}^s)^{1-\mu} (C_{ss} - \text{pers} H_{ss})^\mu \right)^{-\eta} = 0 \quad (6.7)$$

$$-\lambda_{ss}^{\text{CONSUMER}^2} + \mu (1 - L_{ss}^s)^{1-\mu} (C_{ss} - \text{pers} H_{ss})^{-1+\mu} \left((1 - L_{ss}^s)^{1-\mu} (C_{ss} - \text{pers} H_{ss})^\mu \right)^{-\eta} - \beta \mu \text{pers} (1 - L_{ss}^s)^{1-\mu} (C_{ss} - \text{pers} H_{ss})^{-1+\mu} \left((1 - L_{ss}^s)^{1-\mu} (C_{ss} - \text{pers} H_{ss})^\mu \right)^{-\eta} = 0 \quad (6.8)$$

$$-C_{ss} - I_{ss} + Y_{ss} = 0 \quad (6.9)$$

$$I_{ss} - K_{ss}^s + K_{ss}^s (1 - \delta) = 0 \quad (6.10)$$

$$U_{ss} - \beta U_{ss} - (1 - \eta)^{-1} \left((1 - L_{ss}^s)^{1-\mu} (C_{ss} - \text{pers} H_{ss})^\mu \right)^{1-\eta} = 0 \quad (6.11)$$

7 Calibrating equations

$$-0.36 Y_{ss} + r_{ss} K_{ss}^s = 0 \quad (7.1)$$

8 Parameter settings

$$\beta = 0.99 \tag{8.1}$$

$$\delta = 0.025 \tag{8.2}$$

$$\eta = 2 \tag{8.3}$$

$$\mu = 0.3 \tag{8.4}$$

$$pers = 0.57 \tag{8.5}$$

$$\phi = 0.95 \tag{8.6}$$

9 Steady-state values

| | Steady-state value |
|-------------------------------|--------------------|
| $\lambda^{\text{CONSUMER}^2}$ | 0.7116 |
| r | 0.0351 |
| C | 0.7494 |
| H | 0.7494 |
| I | 0.2584 |
| K^s | 10.3356 |
| L^s | 0.2721 |
| U | -175.4236 |
| W | 2.3706 |
| Y | 1.0078 |
| Z | 1 |

10 The solution of the 1st order perturbation

Matrix P

$$\begin{matrix} C_t \\ K_t^s \\ Z_t \end{matrix} \begin{pmatrix} C_{t-1} & K_{t-1}^s & Z_{t-1} \\ 0.5544 & 0.0151 & 0.1764 \\ -0.5092 & 0.9817 & 1.1759 \\ 0 & 0 & 0.95 \end{pmatrix}$$

Matrix Q

$$\begin{matrix} C \\ K^s \\ Z \end{matrix} \begin{pmatrix} 0.1857 \\ 1.2377 \\ 1 \end{pmatrix} \epsilon^Z$$

Matrix R

$$\begin{matrix} \lambda_t^{\text{CONSUMER}^2} \\ r_t \\ H_t \\ I_t \\ L_t^s \\ U_t \\ W_t \\ Y_t \end{matrix} \begin{pmatrix} C_{t-1} & K_{t-1}^s & Z_{t-1} \\ 0.0599 & -0.0494 & -0.3592 \\ 0.0016 & -0.0026 & 0.0471 \\ 1 & 0 & 0 \\ -0.5092 & 0.0067 & 1.1759 \\ 0.0191 & -0.0056 & 0.1666 \\ -0.9309 & 0.7188 & 11.4498 \\ -0.0598 & 0.1002 & 1.7296 \\ 0.0452 & 0.0218 & 1.3522 \end{pmatrix}$$

Matrix S

$$\begin{matrix} \lambda^{\text{CONSUMER}^2} \\ r \\ H \\ I \\ L^s \\ U \\ W \\ Y \end{matrix} \begin{pmatrix} -0.3781 \\ 0.0496 \\ 0 \\ 1.2377 \\ 0.1753 \\ 12.0524 \\ 1.8206 \\ 1.4234 \end{pmatrix} \epsilon^Z$$

11 Model statistics

11.1 Basic statistics

| | Steady-state value | Std. dev. | Variance | Loglin |
|-------|--------------------|-----------|----------|--------|
| r | 0.0351 | 0.0046 | 0 | N |
| C | 0.7494 | 0.0333 | 0.0011 | N |
| H | 0.7494 | 0.0333 | 0.0011 | N |
| I | 0.2584 | 0.1077 | 0.0116 | N |
| K^s | 10.3356 | 0.3633 | 0.132 | N |
| L^s | 0.2721 | 0.0164 | 0.0003 | N |
| U | -175.4236 | 1.1325 | 1.2825 | N |
| W | 2.3706 | 0.1719 | 0.0295 | N |
| Y | 1.0078 | 0.1325 | 0.0175 | N |
| Z | 1 | 0.0922 | 0.0085 | N |

11.2 Correlation matrix

| | r | C | H | I | K^s | L^s | U | W | Y | Z |
|-------|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| r | 1 | 0.623 | 0.263 | 0.993 | 0.14 | 0.994 | 0.918 | 0.92 | 0.965 | 0.98 |
| C | | 1 | 0.911 | 0.674 | 0.826 | 0.701 | 0.867 | 0.865 | 0.799 | 0.758 |
| H | | | 1 | 0.315 | 0.899 | 0.358 | 0.584 | 0.582 | 0.485 | 0.429 |
| I | | | | 1 | 0.232 | 0.997 | 0.95 | 0.951 | 0.983 | 0.992 |
| K^s | | | | | 1 | 0.243 | 0.521 | 0.516 | 0.396 | 0.332 |
| L^s | | | | | | 1 | 0.954 | 0.956 | 0.987 | 0.996 |
| U | | | | | | | 1 | 1 | 0.99 | 0.978 |
| W | | | | | | | | 1 | 0.991 | 0.979 |
| Y | | | | | | | | | 1 | 0.998 |
| Z | | | | | | | | | | 1 |

11.3 Cross correlations with the reference variable (Y)

| | $\sigma[\cdot]$ rel. to $\sigma[Y]$ | Y_{t-5} | Y_{t-4} | Y_{t-3} | Y_{t-2} | Y_{t-1} | Y_t | Y_{t+1} | Y_{t+2} | Y_{t+3} | Y_{t+4} | Y_{t+5} |
|---------|-------------------------------------|-----------|-----------|-----------|-----------|-----------|-------|-----------|-----------|-----------|-----------|-----------|
| r_t | 0.035 | 0.126 | 0.245 | 0.388 | 0.556 | 0.749 | 0.965 | 0.613 | 0.331 | 0.109 | -0.059 | -0.182 |
| C_t | 0.251 | -0.244 | -0.131 | 0.025 | 0.228 | 0.485 | 0.799 | 0.867 | 0.804 | 0.677 | 0.527 | 0.375 |
| H_t | 0.251 | -0.32 | -0.244 | -0.131 | 0.025 | 0.228 | 0.485 | 0.799 | 0.867 | 0.804 | 0.677 | 0.527 |
| I_t | 0.813 | 0.069 | 0.193 | 0.345 | 0.527 | 0.74 | 0.983 | 0.622 | 0.349 | 0.143 | -0.011 | -0.122 |
| K_t^s | 2.742 | -0.459 | -0.39 | -0.278 | -0.115 | 0.107 | 0.396 | 0.57 | 0.66 | 0.685 | 0.665 | 0.612 |
| L_t^s | 0.124 | 0.076 | 0.2 | 0.352 | 0.534 | 0.746 | 0.987 | 0.668 | 0.402 | 0.185 | 0.016 | -0.113 |
| U_t | 8.55 | -0.077 | 0.053 | 0.222 | 0.432 | 0.688 | 0.99 | 0.754 | 0.545 | 0.366 | 0.214 | 0.089 |
| W_t | 1.298 | -0.073 | 0.057 | 0.226 | 0.436 | 0.69 | 0.991 | 0.755 | 0.546 | 0.365 | 0.211 | 0.085 |
| Y_t | 1 | -0.005 | 0.124 | 0.286 | 0.486 | 0.724 | 1 | 0.724 | 0.486 | 0.286 | 0.124 | -0.005 |
| Z_t | 0.696 | 0.028 | 0.155 | 0.314 | 0.507 | 0.735 | 0.998 | 0.698 | 0.446 | 0.24 | 0.076 | -0.051 |

11.4 Autocorrelations

| | Lag 1 | Lag 2 | Lag 3 | Lag 4 | Lag 5 |
|-------|-------|-------|-------|-------|--------|
| r | 0.707 | 0.462 | 0.261 | 0.101 | 0 |
| C | 0.911 | 0.738 | 0.537 | 0.337 | 0.156 |
| H | 0.911 | 0.738 | 0.537 | 0.337 | 0.156 |
| I | 0.667 | 0.413 | 0.217 | 0.069 | -0.043 |
| K^s | 0.955 | 0.851 | 0.71 | 0.55 | 0.384 |
| L^s | 0.716 | 0.473 | 0.271 | 0.108 | -0.02 |
| U | 0.737 | 0.509 | 0.316 | 0.155 | 0.025 |
| W | 0.74 | 0.512 | 0.317 | 0.155 | 0.024 |
| Y | 0.724 | 0.486 | 0.286 | 0.124 | -0.005 |
| Z | 0.713 | 0.471 | 0.271 | 0.11 | -0.016 |

12 Impulse response functions

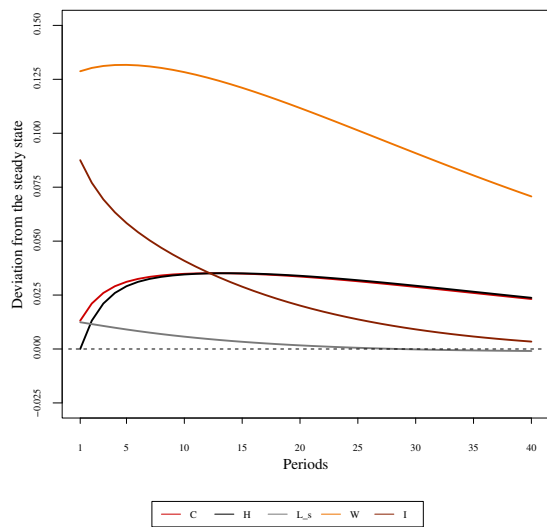


Figure 1: Impulse responses (C, H, L^s, W, I) to ϵ^Z shock

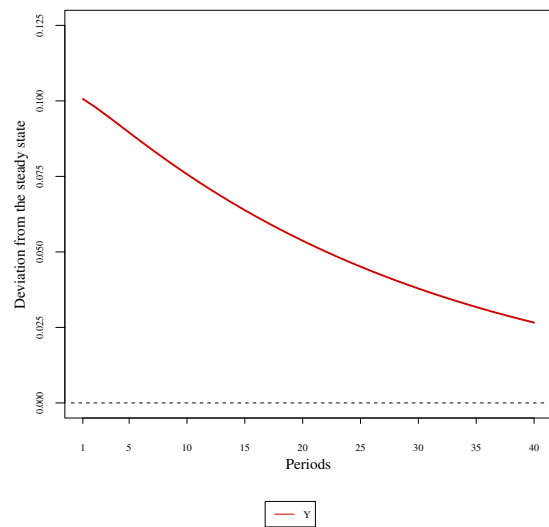


Figure 2: Impulse response (Y) to ϵ^Z shock