

## 1 CONSUMER

### 1.1 Optimisation problem

$$\max_{K_t^s, C_t, L_t^s, I_t} U_t = \beta \mathbf{E}_t [U_{t+1}] + (1 - \eta)^{-1} \left( C_t^\mu (1 - L_t^s)^{1-\mu} \right)^{1-\eta} \quad (1.1)$$

s.t. :

$$C_t + I_t = \pi_t + K_{t-1}^s r_t + L_t^s W_t - \psi K_{t-1}^s \left( -\delta + K_{t-1}^{s-1} I_t \right)^2 \quad (\lambda_t^c) \quad (1.2)$$

$$K_t^s = I_t + K_{t-1}^s (1 - \delta) \quad (\lambda_t^{\text{CONSUMER}^2}) \quad (1.3)$$

### 1.2 First order conditions

$$-\lambda_t^{\text{CONSUMER}^2} + \beta \left( (1 - \delta) \mathbf{E}_t \left[ \lambda_{t+1}^{\text{CONSUMER}^2} \right] + \mathbf{E}_t \left[ \lambda_{t+1}^c \left( r_{t+1} - \psi \left( -\delta + K_t^{s-1} I_{t+1} \right)^2 + 2\psi K_t^{s-1} I_{t+1} \left( -\delta + K_t^{s-1} I_{t+1} \right) \right) \right] \right) = 0 \quad (K_t^s) \quad (1.4)$$

$$-\lambda_t^c + \mu C_t^{-1+\mu} (1 - L_t^s)^{1-\mu} \left( C_t^\mu (1 - L_t^s)^{1-\mu} \right)^{-\eta} = 0 \quad (C_t) \quad (1.5)$$

$$\lambda_t^c W_t + (-1 + \mu) C_t^\mu (1 - L_t^s)^{-\mu} \left( C_t^\mu (1 - L_t^s)^{1-\mu} \right)^{-\eta} = 0 \quad (L_t^s) \quad (1.6)$$

$$\lambda_t^{\text{CONSUMER}^2} + \lambda_t^c \left( -1 - 2\psi \left( -\delta + K_{t-1}^{s-1} I_t \right) \right) = 0 \quad (I_t) \quad (1.7)$$

## 2 FIRM

### 2.1 Optimisation problem

$$\max_{K_t^d, L_t^d, Y_t, \pi_t} \Pi_t = \pi_t \quad (2.1)$$

s.t. :

$$Y_t = Z_t K_t^{d\alpha} L_t^{d1-\alpha} \quad (\lambda_t^{\text{FIRM}^1}) \quad (2.2)$$

$$\pi_t = Y_t - L_t^d W_t - r_t K_t^d \quad (\lambda_t^{\text{FIRM}^2}) \quad (2.3)$$

## 2.2 First order conditions

$$-\lambda_t^{\text{FIRM}^2} r_t + \alpha \lambda_t^{\text{FIRM}^1} Z_t K_t^{\text{d}-1+\alpha} L_t^{\text{d}1-\alpha} = 0 \quad (K_t^{\text{d}}) \quad (2.4)$$

$$-\lambda_t^{\text{FIRM}^2} W_t + \lambda_t^{\text{FIRM}^1} Z_t (1 - \alpha) K_t^{\text{d}\alpha} L_t^{\text{d}-\alpha} = 0 \quad (L_t^{\text{d}}) \quad (2.5)$$

$$-\lambda_t^{\text{FIRM}^1} + \lambda_t^{\text{FIRM}^2} = 0 \quad (Y_t) \quad (2.6)$$

$$1 - \lambda_t^{\text{FIRM}^2} = 0 \quad (\pi_t) \quad (2.7)$$

## 2.3 First order conditions after reduction

$$-r_t + \alpha Z_t K_t^{\text{d}-1+\alpha} L_t^{\text{d}1-\alpha} = 0 \quad (K_t^{\text{d}}) \quad (2.8)$$

$$-W_t + Z_t (1 - \alpha) K_t^{\text{d}\alpha} L_t^{\text{d}-\alpha} = 0 \quad (L_t^{\text{d}}) \quad (2.9)$$

# 3 EQUILIBRIUM

## 3.1 Identities

$$K_t^{\text{d}} = K_{t-1}^{\text{s}} \quad (3.1)$$

$$L_t^{\text{d}} = L_t^{\text{s}} \quad (3.2)$$

# 4 EXOG

## 4.1 Identities

$$Z_t = e^{\epsilon_t^Z + \phi \log Z_{t-1}} \quad (4.1)$$

## 5 Equilibrium relationships (after reduction)

$$-r_t + \alpha Z_t K_{t-1}^{\text{s}-1+\alpha} L_t^{\text{s}1-\alpha} = 0 \quad (5.1)$$

$$-W_t + Z_t (1 - \alpha) K_{t-1}^{\text{s}\alpha} L_t^{\text{s}-\alpha} = 0 \quad (5.2)$$

$$-Y_t + Z_t K_{t-1}^s \alpha L_t^{s1-\alpha} = 0 \quad (5.3)$$

$$-Z_t + e^{\epsilon_t^Z + \phi \log Z_{t-1}} = 0 \quad (5.4)$$

$$\beta \left( \mu \mathbb{E}_t \left[ \left( r_{t+1} - \psi \left( -\delta + K_t^{s-1} I_{t+1} \right)^2 + 2\psi K_t^{s-1} I_{t+1} \left( -\delta + K_t^{s-1} I_{t+1} \right) \right) C_{t+1}^{-1+\mu} (1 - L_{t+1}^s)^{1-\mu} \left( C_{t+1}^\mu (1 - L_{t+1}^s)^{1-\mu} \right)^{-\eta} \right] - \mu (1 - \delta) \mathbb{E}_t \left[ \left( -1 - 2\psi \left( -\delta + K_t^{s-1} I_{t+1} \right) \right) C_{t+1}^{-1} \right] \right) \quad (5.5)$$

$$(-1 + \mu) C_t^\mu (1 - L_t^s)^{-\mu} \left( C_t^\mu (1 - L_t^s)^{1-\mu} \right)^{-\eta} + \mu W_t C_t^{-1+\mu} (1 - L_t^s)^{1-\mu} \left( C_t^\mu (1 - L_t^s)^{1-\mu} \right)^{-\eta} = 0 \quad (5.6)$$

$$I_t - K_t^s + K_{t-1}^s (1 - \delta) = 0 \quad (5.7)$$

$$U_t - \beta \mathbb{E}_t [U_{t+1}] - (1 - \eta)^{-1} \left( C_t^\mu (1 - L_t^s)^{1-\mu} \right)^{1-\eta} = 0 \quad (5.8)$$

$$-C_t - I_t + Y_t - \psi K_{t-1}^s \left( -\delta + K_{t-1}^{s-1} I_t \right)^2 = 0 \quad (5.9)$$

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## 6 Steady state relationships (after reduction)

$$-r_{ss} + \alpha Z_{ss} K_{ss}^{s-1+\alpha} L_{ss}^{s1-\alpha} = 0 \quad (6.1)$$

$$-W_{ss} + Z_{ss} (1 - \alpha) K_{ss}^{s\alpha} L_{ss}^{s-\alpha} = 0 \quad (6.2)$$

$$-Y_{ss} + Z_{ss} K_{ss}^{s\alpha} L_{ss}^{s1-\alpha} = 0 \quad (6.3)$$

$$-Z_{ss} + e^{\phi \log Z_{ss}} = 0 \quad (6.4)$$

$$\beta \left( \mu \left( r_{ss} - \psi \left( -\delta + I_{ss} K_{ss}^{s-1} \right)^2 + 2\psi I_{ss} K_{ss}^{s-1} \left( -\delta + I_{ss} K_{ss}^{s-1} \right) \right) C_{ss}^{-1+\mu} (1 - L_{ss}^s)^{1-\mu} \left( C_{ss}^\mu (1 - L_{ss}^s)^{1-\mu} \right)^{-\eta} - \mu \left( -1 - 2\psi \left( -\delta + I_{ss} K_{ss}^{s-1} \right) \right) (1 - \delta) C_{ss}^{-1+\mu} (1 - L_{ss}^s)^{1-\mu} \left( C_{ss}^\mu (1 - L_{ss}^s)^{1-\mu} \right)^{-\eta} \right) \quad (6.5)$$

$$(-1 + \mu) C_{ss}^\mu (1 - L_{ss}^s)^{-\mu} \left( C_{ss}^\mu (1 - L_{ss}^s)^{1-\mu} \right)^{-\eta} + \mu W_{ss} C_{ss}^{-1+\mu} (1 - L_{ss}^s)^{1-\mu} \left( C_{ss}^\mu (1 - L_{ss}^s)^{1-\mu} \right)^{-\eta} = 0 \quad (6.6)$$

$$I_{ss} - K_{ss}^s + K_{ss}^s (1 - \delta) = 0 \quad (6.7)$$

$$U_{ss} - \beta U_{ss} - (1 - \eta)^{-1} \left( C_{ss}^\mu (1 - L_{ss}^s)^{1-\mu} \right)^{1-\eta} = 0 \quad (6.8)$$

$$-C_{ss} - I_{ss} + Y_{ss} - \psi K_{ss}^s \left( -\delta + I_{ss} K_{ss}^s \right)^2 = 0 \quad (6.9)$$

## 7 Calibrating equations

$$-0.36Y_{ss} + r_{ss}K_{ss}^s = 0 \quad (7.1)$$

## 8 Parameter settings

$$\beta = 0.99 \quad (8.1)$$

$$\delta = 0.025 \quad (8.2)$$

$$\eta = 2 \quad (8.3)$$

$$\mu = 0.3 \quad (8.4)$$

$$\phi = 0.95 \quad (8.5)$$

$$\psi = 0.8 \quad (8.6)$$

## 9 Steady-state values

	Steady-state value
$r$	0.0351
$C$	0.7422
$I$	0.2559
$K^s$	10.2368
$L^s$	0.2695
$U$	-136.2372
$W$	2.3706
$Y$	0.9981
$Z$	1

## 10 The solution of the 1st order perturbation

Matrix  $P$

$$\begin{matrix} & K_{t-1}^s & Z_{t-1} \\ K_t^s & \left( \begin{array}{cc} 0.9658 & 0.0863 \\ 0 & 0.95 \end{array} \right) \\ Z_t & \end{matrix}$$

Matrix  $Q$

$$\begin{matrix} & \epsilon^Z \\ K^s & \left( \begin{array}{c} 0.0908 \\ 1 \end{array} \right) \\ Z & \end{matrix}$$

Matrix  $R$

$$\begin{matrix} & K_{t-1}^s & Z_{t-1} \\ r_t & \left( \begin{array}{cc} -0.7408 & 1.2972 \\ 0.4748 & 0.5545 \\ -0.3661 & 3.4511 \\ -0.1575 & 0.5426 \\ 0.0418 & 0.0644 \\ 0.4167 & 0.7547 \\ 0.2592 & 1.2972 \end{array} \right) \\ C_t & \\ I_t & \\ L_t^s & \\ U_t & \\ W_t & \\ Y_t & \end{matrix}$$

Matrix  $S$

$$\begin{matrix} & \epsilon^Z \\ r & \left( \begin{array}{c} 1.3655 \\ 0.5837 \\ 3.6328 \\ 0.5711 \\ 0.0678 \\ 0.7944 \\ 1.3655 \end{array} \right) \\ C & \\ I & \\ L^s & \\ U & \\ W & \\ Y & \end{matrix}$$

## 11 Model statistics

### 11.1 Basic statistics

	Steady-state value	Std. dev.	Variance	Loglin
$r$	0.0351	0.1814	0.0329	Y
$C$	0.7422	0.0783	0.0061	Y
$I$	0.2559	0.4741	0.2248	Y
$K^s$	10.2368	0.0422	0.0018	Y
$L^s$	0.2695	0.0749	0.0056	Y
$U$	-136.2372	0.009	0.0001	Y
$W$	2.3706	0.1047	0.011	Y
$Y$	0.9981	0.1781	0.0317	Y
$Z$	1	0.1303	0.017	Y

### 11.2 Correlation matrix

	$r$	$C$	$I$	$K^s$	$L^s$	$U$	$W$	$Y$	$Z$
$r$	1	0.908	0.99	0.09	0.996	0.932	0.942	0.973	0.985
$C$		1	0.958	0.498	0.94	0.998	0.996	0.981	0.967
$I$			1	0.228	0.998	0.974	0.98	0.996	0.999
$K^s$				1	0.173	0.445	0.418	0.319	0.26
$L^s$					1	0.959	0.967	0.989	0.996
$U$						1	1	0.991	0.981
$W$							1	0.994	0.986
$Y$								1	0.998
$Z$									1

### 11.3 Cross correlations with the reference variable ( $Y$ )

	$\sigma[\cdot]$ rel. to $\sigma[Y]$	$Y_{t-5}$	$Y_{t-4}$	$Y_{t-3}$	$Y_{t-2}$	$Y_{t-1}$	$Y_t$	$Y_{t+1}$	$Y_{t+2}$	$Y_{t+3}$	$Y_{t+4}$	$Y_{t+5}$
$r_t$	1.018	0.109	0.228	0.373	0.545	0.745	0.973	0.631	0.353	0.132	-0.037	-0.161
$C_t$	0.439	-0.107	0.021	0.189	0.402	0.665	0.981	0.761	0.564	0.392	0.245	0.121
$I_t$	2.662	0.039	0.164	0.319	0.508	0.733	0.996	0.688	0.431	0.222	0.057	-0.07
$K_t^s$	0.237	-0.48	-0.422	-0.321	-0.17	0.04	0.319	0.504	0.612	0.66	0.659	0.623
$L_t^s$	0.42	0.067	0.19	0.341	0.524	0.74	0.989	0.666	0.401	0.187	0.019	-0.107
$U_t$	0.05	-0.077	0.052	0.218	0.428	0.684	0.991	0.751	0.54	0.359	0.207	0.081
$W_t$	0.588	-0.062	0.066	0.232	0.439	0.692	0.994	0.745	0.528	0.343	0.188	0.062
$Y_t$	1	-0.008	0.119	0.28	0.479	0.718	1	0.718	0.479	0.28	0.119	-0.008
$Z_t$	0.732	0.023	0.148	0.306	0.499	0.729	0.998	0.699	0.448	0.242	0.078	-0.049

### 11.4 Autocorrelations

	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5
$r$	0.71	0.466	0.266	0.104	-0.022
$C$	0.745	0.521	0.329	0.169	0.038
$I$	0.712	0.468	0.268	0.107	-0.019
$K^s$	0.96	0.863	0.728	0.572	0.408
$L^s$	0.71	0.466	0.265	0.103	-0.022
$U$	0.735	0.505	0.311	0.15	0.02
$W$	0.73	0.498	0.303	0.142	0.013
$Y$	0.718	0.479	0.28	0.119	-0.008
$Z$	0.713	0.471	0.271	0.11	-0.016

## 12 Impulse response functions

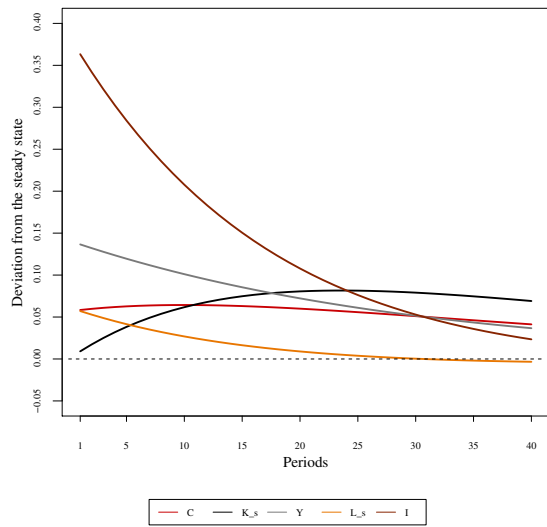


Figure 1: Impulse responses ( $C, K^s, Y, L^s, I$ ) to  $\epsilon^Z$  shock

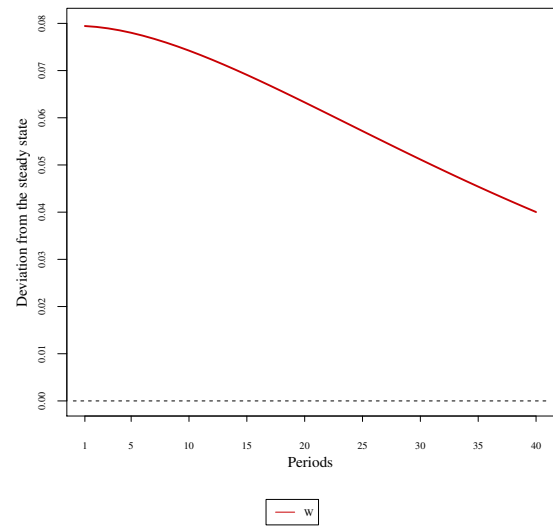


Figure 2: Impulse response ( $W$ ) to  $\epsilon^Z$  shock