

1 CONSUMER

1.1 Optimisation problem

$$\max_{C_t, L_t^s} U_t = \beta E_t [U_{t+1}] + (1 - \eta)^{-1} \left(C_t^\mu (1 - L_t^s)^{1-\mu} \right)^{1-\eta} \quad (1.1)$$

s.t. :

$$C_t P_t^{\text{FIN}} = \pi_t + \pi_t^{\text{ps}} + L_t^s W_t \quad (\lambda_t^c) \quad (1.2)$$

1.2 First order conditions

$$\beta - \lambda_t^U = 0 \quad (U_t) \quad (1.3)$$

$$-\lambda_t^c P_t^{\text{FIN}} + \mu C_t^{-1+\mu} (1 - L_t^s)^{1-\mu} \left(C_t^\mu (1 - L_t^s)^{1-\mu} \right)^{-\eta} = 0 \quad (C_t) \quad (1.4)$$

$$\lambda_t^c W_t + (-1 + \mu) C_t^\mu (1 - L_t^s)^{-\mu} \left(C_t^\mu (1 - L_t^s)^{1-\mu} \right)^{-\eta} = 0 \quad (L_t^s) \quad (1.5)$$

2 INTERMEDIATE FIRM

2.1 Optimisation problem

$$\max_{K_t, L_t^d, Y_t, I_t, \pi_t} \Pi_t = \pi_t + \lambda_t^{c-1} E_t [\lambda_{t+1}^U \Pi_{t+1}] \quad (2.1)$$

s.t. :

$$\pi_t = -I_t - L_t^d W_t + P_t Y_t \quad \left(\lambda_t^{\text{INTERMEDIATE}^{\text{FIRM}^1}} \right) \quad (2.2)$$

$$Y_t = K_{t-1}^\alpha (L_t^d Z_t)^{1-\alpha} \quad \left(\lambda_t^{\text{INTERMEDIATE}^{\text{FIRM}^2}} \right) \quad (2.3)$$

$$K_t = I_t + K_{t-1} (1 - \delta) \quad \left(\lambda_t^{\text{INTERMEDIATE}^{\text{FIRM}^3}} \right) \quad (2.4)$$

2.2 First order conditions

$$-\lambda_t^{\text{INTERMEDIATE}^{\text{FIRM}^{\text{II}}}} + \lambda_{t-1}^{\text{c}}^{-1} \lambda_t^{\text{c}} \lambda_t^{\text{U}} = 0 \quad (\Pi_t) \quad (2.5)$$

$$-\lambda_t^{\text{INTERMEDIATE}^{\text{FIRM}^{\text{III}}}} + \text{E}_t \left[\lambda_{t+1}^{\text{INTERMEDIATE}^{\text{FIRM}^{\text{II}}}} \left(\lambda_{t+1}^{\text{INTERMEDIATE}^{\text{FIRM}^{\text{III}}}} (1 - \delta) + \alpha \lambda_{t+1}^{\text{INTERMEDIATE}^{\text{FIRM}^{\text{II}}}} K_t^{-1+\alpha} (L_{t+1}^{\text{d}} Z_{t+1})^{1-\alpha} \right) \right] = 0 \quad (K_t) \quad (2.6)$$

$$-\lambda_t^{\text{INTERMEDIATE}^{\text{FIRM}^{\text{I}}}} W_t + \lambda_t^{\text{INTERMEDIATE}^{\text{FIRM}^{\text{II}}}} Z_t (1 - \alpha) K_{t-1}^{\alpha} (L_t^{\text{d}} Z_t)^{-\alpha} = 0 \quad (L_t^{\text{d}}) \quad (2.7)$$

$$-\lambda_t^{\text{INTERMEDIATE}^{\text{FIRM}^{\text{II}}}} + \lambda_t^{\text{INTERMEDIATE}^{\text{FIRM}^{\text{I}}}} P_t = 0 \quad (Y_t) \quad (2.8)$$

$$-\lambda_t^{\text{INTERMEDIATE}^{\text{FIRM}^{\text{I}}}} + \lambda_t^{\text{INTERMEDIATE}^{\text{FIRM}^{\text{III}}}} = 0 \quad (I_t) \quad (2.9)$$

$$1 - \lambda_t^{\text{INTERMEDIATE}^{\text{FIRM}^{\text{I}}}} = 0 \quad (\pi_t) \quad (2.10)$$

2.3 First order conditions after reduction

$$-\lambda_t^{\text{INTERMEDIATE}^{\text{FIRM}^{\text{II}}}} + \lambda_{t-1}^{\text{c}}^{-1} \lambda_t^{\text{c}} \lambda_t^{\text{U}} = 0 \quad (\Pi_t) \quad (2.11)$$

$$-1 + \text{E}_t \left[\lambda_{t+1}^{\text{INTERMEDIATE}^{\text{FIRM}^{\text{II}}}} \left(1 - \delta + \alpha \lambda_{t+1}^{\text{INTERMEDIATE}^{\text{FIRM}^{\text{II}}}} K_t^{-1+\alpha} (L_{t+1}^{\text{d}} Z_{t+1})^{1-\alpha} \right) \right] = 0 \quad (K_t) \quad (2.12)$$

$$-W_t + \lambda_t^{\text{INTERMEDIATE}^{\text{FIRM}^{\text{II}}}} Z_t (1 - \alpha) K_{t-1}^{\alpha} (L_t^{\text{d}} Z_t)^{-\alpha} = 0 \quad (L_t^{\text{d}}) \quad (2.13)$$

$$-\lambda_t^{\text{INTERMEDIATE}^{\text{FIRM}^{\text{II}}}} + P_t = 0 \quad (Y_t) \quad (2.14)$$

3 PRICE SETTING

3.1 Optimisation problem

$$\max_{\pi_t^{\text{ps}}, Y_t^{\text{MON}}, P_t^{\text{MON}}} \Pi_t^{\text{PS}} = \pi_t^{\text{ps}} \quad (3.1)$$

s.t. :

$$\pi_t^{\text{ps}} = Y_t^{\text{MON}} (-P_t + P_t^{\text{MON}}) \left(\lambda_t^{\text{PRICE}^{\text{SETTING}^{\text{I}}}} \right) \quad (3.2)$$

$$Y_t^{\text{MON}} = Y_t^{\text{FIN}} \left(P_t^{\text{FIN}^{-1}} P_t^{\text{MON}} \right)^{-\rho} \left(\lambda_t^{\text{PRICE}^{\text{SETTING}^{\text{II}}}} \right) \quad (3.3)$$

3.2 First order conditions

$$1 - \lambda_t^{\text{PRICESETTING}^1} = 0 \quad (\pi_t^{\text{PS}}) \quad (3.4)$$

$$-\lambda_t^{\text{PRICESETTING}^2} + \lambda_t^{\text{PRICESETTING}^1} (-P_t + P_t^{\text{MON}}) = 0 \quad (Y_t^{\text{MON}}) \quad (3.5)$$

$$\lambda_t^{\text{PRICESETTING}^1} Y_t^{\text{MON}} - \rho \lambda_t^{\text{PRICESETTING}^2} P_t^{\text{FIN}-1} Y_t^{\text{FIN}} (P_t^{\text{FIN}-1} P_t^{\text{MON}})^{-1-\rho} = 0 \quad (P_t^{\text{MON}}) \quad (3.6)$$

3.3 First order conditions after reduction

$$Y_t^{\text{MON}} - \rho P_t^{\text{FIN}-1} Y_t^{\text{FIN}} (-P_t + P_t^{\text{MON}}) (P_t^{\text{FIN}-1} P_t^{\text{MON}})^{-1-\rho} = 0 \quad (P_t^{\text{MON}}) \quad (3.7)$$

4 FINAL FIRM

4.1 Identities

$$Y_t^{\text{FIN}} = Y_t^{\text{MON}} \quad (4.1)$$

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5 EQUILIBRIUM

5.1 Identities

$$L_t^{\text{d}} = L_t^{\text{s}} \quad (5.1)$$

$$P_t^{\text{FIN}} = 1 \quad (5.2)$$

$$Y_t^{\text{MON}} = Y_t \quad (5.3)$$

6 EXOG

6.1 Identities

$$Z_t = e^{\epsilon_t^Z + \phi \log Z_{t-1}} \quad (6.1)$$

7 Equilibrium relationships (after reduction)

$$-1+\beta (C_t^{-1+\mu})^{-1} \left((1-L_t^s)^{1-\mu} \right)^{-1} \left((C_t^\mu (1-L_t^s)^{1-\mu})^{-\eta} \right)^{-1} \mathbb{E}_t \left[\left(1-\delta + \alpha P_{t+1} K_t^{-1+\alpha} (L_{t+1}^s Z_{t+1})^{1-\alpha} \right) C_{t+1}^{-1+\mu} (1-L_{t+1}^s)^{1-\mu} \left(C_{t+1}^\mu (1-L_{t+1}^s)^{1-\mu} \right)^{-\eta} \right] = 0 \quad (7.1)$$

$$-\pi_t^{\text{ps}} + \Pi_t^{\text{PS}} = 0 \quad (7.2)$$

$$-\pi_t^{\text{ps}} + Y_t (-P_t + P_t^{\text{MON}}) = 0 \quad (7.3)$$

$$-W_t + P_t Z_t (1-\alpha) K_{t-1}^\alpha (L_t^s Z_t)^{-\alpha} = 0 \quad (7.4)$$

$$-Y_t + Y_t P_t^{\text{MON}^{-\rho}} = 0 \quad (7.5)$$

$$-Y_t + K_{t-1}^\alpha (L_t^s Z_t)^{1-\alpha} = 0 \quad (7.6)$$

$$Y_t - \rho Y_t (-P_t + P_t^{\text{MON}}) P_t^{\text{MON}^{-1-\rho}} = 0 \quad (7.7)$$

$$-Z_t + e^{\epsilon_t^Z + \phi \log Z_{t-1}} = 0 \quad (7.8)$$

$$(-1+\mu) C_t^\mu (1-L_t^s)^{-\mu} \left(C_t^\mu (1-L_t^s)^{1-\mu} \right)^{-\eta} + \mu W_t C_t^{-1+\mu} (1-L_t^s)^{1-\mu} \left(C_t^\mu (1-L_t^s)^{1-\mu} \right)^{-\eta} = 0 \quad (7.9)$$

$$-\pi_t + \Pi_t - \beta (C_t^{-1+\mu})^{-1} \left((1-L_t^s)^{1-\mu} \right)^{-1} \left((C_t^\mu (1-L_t^s)^{1-\mu})^{-\eta} \right)^{-1} \mathbb{E}_t \left[\Pi_{t+1} C_{t+1}^{-1+\mu} (1-L_{t+1}^s)^{1-\mu} \left(C_{t+1}^\mu (1-L_{t+1}^s)^{1-\mu} \right)^{-\eta} \right] = 0 \quad (7.10)$$

$$I_t - K_t + K_{t-1} (1-\delta) = 0 \quad (7.11)$$

$$U_t - \beta \mathbb{E}_t [U_{t+1}] - (1-\eta)^{-1} \left(C_t^\mu (1-L_t^s)^{1-\mu} \right)^{1-\eta} = 0 \quad (7.12)$$

$$-\pi_t - I_t - L_t^s W_t + P_t Y_t = 0 \quad (7.13)$$

$$\pi_t + \pi_t^{\text{ps}} - C_t + L_t^s W_t = 0 \quad (7.14)$$

8 Steady state relationships (after reduction)

$$-1 + \beta \left(1 - \delta + \alpha P_{ss} K_{ss}^{-1+\alpha} (L_{ss}^s Z_{ss})^{1-\alpha} \right) C_{ss}^{-1+\mu} C_{ss}^{1-\mu} (1 - L_{ss}^s)^{-1+\mu} (1 - L_{ss}^s)^{1-\mu} = 0 \quad (8.1)$$

$$-\pi_{ss}^{\text{ps}} + \Pi_{ss}^{\text{PS}} = 0 \quad (8.2)$$

$$-\pi_{ss}^{\text{ps}} + Y_{ss} (-P_{ss} + P_{ss}^{\text{MON}}) = 0 \quad (8.3)$$

$$-W_{ss} + P_{ss} Z_{ss} (1 - \alpha) K_{ss}^{\alpha} (L_{ss}^s Z_{ss})^{-\alpha} = 0 \quad (8.4)$$

$$-Y_{ss} + Y_{ss} P_{ss}^{\text{MON}^{-\rho}} = 0 \quad (8.5)$$

$$-Y_{ss} + K_{ss}^{\alpha} (L_{ss}^s Z_{ss})^{1-\alpha} = 0 \quad (8.6)$$

$$Y_{ss} - \rho Y_{ss} (-P_{ss} + P_{ss}^{\text{MON}}) P_{ss}^{\text{MON}^{-1-\rho}} = 0 \quad (8.7)$$

$$-Z_{ss} + e^{\phi \log Z_{ss}} = 0 \quad (8.8)$$

$$(-1 + \mu) C_{ss}^{\mu} (1 - L_{ss}^s)^{-\mu} \left(C_{ss}^{\mu} (1 - L_{ss}^s)^{1-\mu} \right)^{-\eta} + \mu W_{ss} C_{ss}^{-1+\mu} (1 - L_{ss}^s)^{1-\mu} \left(C_{ss}^{\mu} (1 - L_{ss}^s)^{1-\mu} \right)^{-\eta} = 0 \quad (8.9)$$

$$-\pi_{ss} + \Pi_{ss} - \beta \Pi_{ss} (1 - L_{ss}^s)^{-1+\mu} (1 - L_{ss}^s)^{1-\mu} = 0 \quad (8.10)$$

$$I_{ss} - K_{ss} + K_{ss} (1 - \delta) = 0 \quad (8.11)$$

$$U_{ss} - \beta U_{ss} - (1 - \eta)^{-1} \left(C_{ss}^{\mu} (1 - L_{ss}^s)^{1-\mu} \right)^{1-\eta} = 0 \quad (8.12)$$

$$-\pi_{ss} - I_{ss} - L_{ss}^s W_{ss} + P_{ss} Y_{ss} = 0 \quad (8.13)$$

$$\pi_{ss} + \pi_{ss}^{\text{ps}} - C_{ss} + L_{ss}^s W_{ss} = 0 \quad (8.14)$$

9 Parameter settings

$$\alpha = 0.33 \tag{9.1}$$

$$\beta = 0.99 \tag{9.2}$$

$$\delta = 0.025 \tag{9.3}$$

$$\eta = 2 \tag{9.4}$$

$$\mu = 0.3 \tag{9.5}$$

$$\phi = 0.95 \tag{9.6}$$

$$\rho = 11 \tag{9.7}$$

10 Steady-state values

	Steady-state value
π	0.0619
π^{PS}	0.0652
C	0.5638
I	0.1532
K	6.1285
L^s	0.2492
P	0.9091
P^{MON}	1
Π	6.1904
Π^{PS}	0.0652
U	-145.144
W	1.7524
Y	0.7171
Z	1

11 The solution of the 1st order perturbation

Matrix P

$$\begin{matrix} K_t & Z_t \end{matrix} \begin{pmatrix} K_{t-1} & Z_{t-1} \\ 0.958 & 0.0744 \\ 0 & 0.95 \end{pmatrix}$$

Matrix Q

$$\begin{matrix} K \\ Z \end{matrix} \begin{pmatrix} \epsilon^Z \\ 0.0783 \\ 1 \end{pmatrix}$$

Matrix R

$$\begin{matrix} \pi_t \\ \pi_t^{\text{PS}} \\ C_t \\ I_t \\ L_t^s \\ P_t \\ P_t^{\text{MON}} \\ \Pi_t \\ \Pi_t^{\text{PS}} \\ U_t \\ W_t \\ Y_t \end{matrix} \begin{pmatrix} K_{t-1} & Z_{t-1} \\ 2.4086 & -4.1777 \\ 0.2085 & 0.9179 \\ 0.45 & 0.3585 \\ -0.6804 & 2.9768 \\ -0.1813 & 0.42 \\ 0 & 0 \\ 0 & 0 \\ 0.9725 & 0.0319 \\ 0.2085 & 0.9179 \\ 0.0343 & 0.0442 \\ 0.3898 & 0.4979 \\ 0.2085 & 0.9179 \end{pmatrix}$$

Matrix S

$$\begin{matrix} \pi \\ \pi^{\text{PS}} \\ C \\ I \\ L^s \\ P \\ P^{\text{MON}} \\ \Pi \\ \Pi^{\text{PS}} \\ U \\ W \\ Y \end{matrix} \begin{pmatrix} -4.3976 \\ 0.9662 \\ 0.3773 \\ 3.1334 \\ 0.4421 \\ 0 \\ 0 \\ 0.0336 \\ 0.9662 \\ 0.0465 \\ 0.5241 \\ 0.9662 \end{pmatrix} \epsilon^Z$$

12 Model statistics

12.1 Basic statistics

	Steady-state value	Std. dev.	Variance	Loglin
C	0.5638	0.5188	0.2691	Y
I	0.1532	4.0905	16.732	Y
K	6.1285	0.3617	0.1308	Y
L^s	0.2492	0.5797	0.336	Y
U	-145.144	0.0619	0.0038	Y
W	1.7524	0.6982	0.4875	Y
Y	0.7171	1.262	1.5927	Y
Z	1	1.3034	1.699	Y

12.2 Correlation matrix

	C	I	K	L^s	U	W	Y	Z
C	1	0.929	0.573	0.908	0.993	0.993	0.967	0.95
I		1	0.229	0.999	0.966	0.965	0.993	0.998
K			1	0.177	0.473	0.474	0.344	0.287
L^s				1	0.951	0.95	0.985	0.994
U					1	1	0.99	0.98
W						1	0.99	0.979
Y							1	0.998
Z								1

12.3 Autocorrelations

	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5
C	0.76	0.545	0.357	0.196	0.063
I	0.71	0.465	0.264	0.103	-0.022
K	0.959	0.861	0.725	0.568	0.403
L^s	0.708	0.463	0.261	0.1	-0.025
U	0.738	0.51	0.316	0.156	0.026
W	0.738	0.51	0.317	0.156	0.026
Y	0.719	0.48	0.281	0.12	-0.007
Z	0.713	0.471	0.271	0.11	-0.016

13 Model statistics

13.1 Basic statistics

	Steady-state value	Std. dev.	Variance	Loglin
<i>C</i>	0.5638	2.7633	7.6358	Y
<i>I</i>	0.1532	8.4869	72.0273	Y
<i>K</i>	6.1285	4.0317	16.255	Y
<i>L^s</i>	0.2492	1.0854	1.1781	Y
<i>U</i>	-145.144	0.2617	0.0685	Y
<i>W</i>	1.7524	2.9592	8.7567	Y
<i>Y</i>	0.7171	3.7017	13.7022	Y
<i>Z</i>	1	3.2026	10.2564	Y

13.2 Correlation matrix

	<i>C</i>	<i>I</i>	<i>K</i>	<i>L^s</i>	<i>U</i>	<i>W</i>	<i>Y</i>	<i>Z</i>
<i>C</i>	1	0.722	0.959	0.498	0.994	0.994	0.941	0.869
<i>I</i>		1	0.495	0.959	0.792	0.791	0.914	0.97
<i>K</i>			1	0.23	0.922	0.923	0.805	0.692
<i>L^s</i>				1	0.588	0.586	0.762	0.862
<i>U</i>					1	1	0.972	0.917
<i>W</i>						1	0.971	0.916
<i>Y</i>							1	0.985
<i>Z</i>								1

13.3 Autocorrelations

	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5
<i>C</i>	0.99	0.98	0.968	0.955	0.941
<i>I</i>	0.929	0.863	0.801	0.743	0.688
<i>K</i>	0.999	0.996	0.991	0.984	0.976
<i>L^s</i>	0.913	0.832	0.756	0.686	0.62
<i>U</i>	0.984	0.967	0.95	0.933	0.915
<i>W</i>	0.984	0.968	0.951	0.933	0.915
<i>Y</i>	0.965	0.932	0.899	0.868	0.837
<i>Z</i>	0.95	0.903	0.857	0.815	0.774

14 Impulse response functions

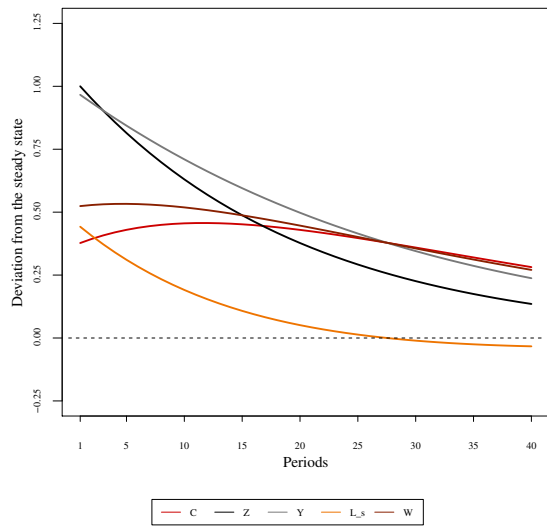


Figure 1: Impulse responses (C, Z, Y, L^s, W) to ϵ^Z shock

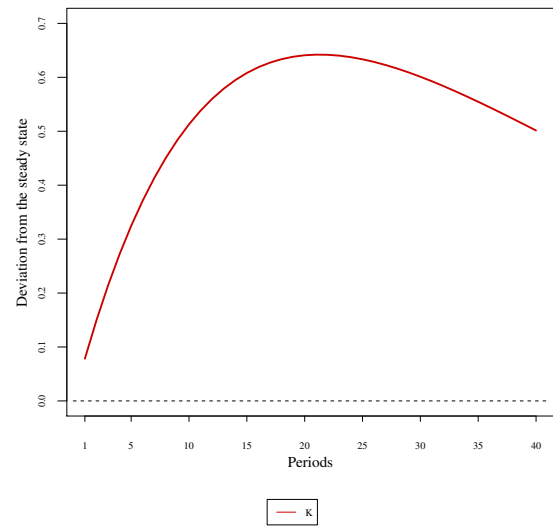


Figure 2: Impulse response (K) to ϵ^Z shock

15 Random path simulation

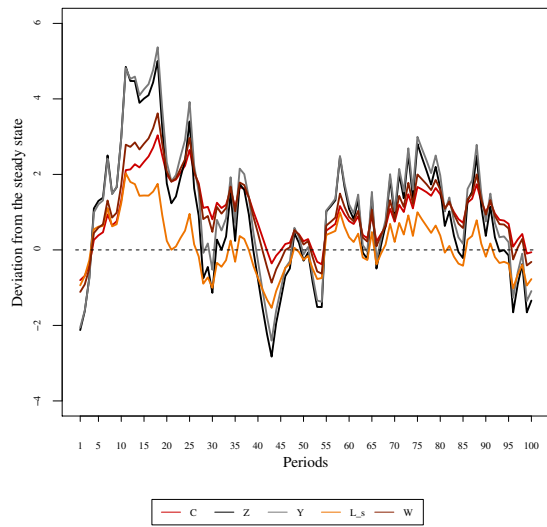


Figure 3: Random path simulation (C, Z, Y, L^s, W)

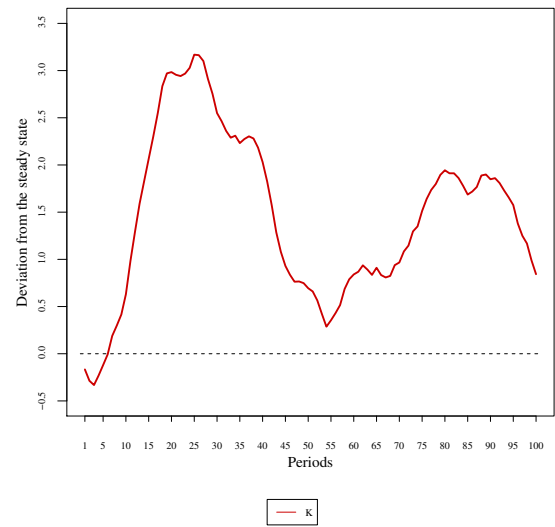


Figure 4: Random path simulation (K)