

1 CONSUMER

1.1 Optimisation problem

$$\max_{K_t^s, C_t, L_t^s, I_t} U_t = \beta E_t [U_{t+1}] + (1 - \eta)^{-1} \left(C_t^\mu (1 - L_t^s)^{1-\mu} \right)^{1-\eta} \quad (1.1)$$

s.t. :

$$C_t + p_t I_t = \pi_t^C + \pi_t^I + K_{t-1}^s r_t + L_t^s W_t \quad \left(\lambda_t^{\text{CONSUMER}^1} \right) \quad (1.2)$$

$$K_t^s = I_t + K_{t-1}^s (1 - \delta) \quad \left(\lambda_t^{\text{CONSUMER}^2} \right) \quad (1.3)$$

1.2 Identities

$$Y_t = C_t + p_t I_t \quad (1.4)$$

1.3 First order conditions

$$-\lambda_t^{\text{CONSUMER}^2} + \beta \left((1 - \delta) E_t \left[\lambda_{t+1}^{\text{CONSUMER}^2} \right] + E_t \left[\lambda_{t+1}^{\text{CONSUMER}^1} r_{t+1} \right] \right) = 0 \quad (K_t^s) \quad (1.5)$$

$$-\lambda_t^{\text{CONSUMER}^1} + \mu C_t^{-1+\mu} (1 - L_t^s)^{1-\mu} \left(C_t^\mu (1 - L_t^s)^{1-\mu} \right)^{-\eta} = 0 \quad (C_t) \quad (1.6)$$

$$\lambda_t^{\text{CONSUMER}^1} W_t + (-1 + \mu) C_t^\mu (1 - L_t^s)^{-\mu} \left(C_t^\mu (1 - L_t^s)^{1-\mu} \right)^{-\eta} = 0 \quad (L_t^s) \quad (1.7)$$

$$\lambda_t^{\text{CONSUMER}^2} - \lambda_t^{\text{CONSUMER}^1} p_t = 0 \quad (I_t) \quad (1.8)$$

2 FIRM C

2.1 Optimisation problem

$$\max_{K_t^{\text{Cd}}, L_t^{\text{Cd}}, C_t^s} \pi_t^C = C_t^s - L_t^{\text{Cd}} W_t - r_t K_t^{\text{Cd}} \quad (2.1)$$

s.t. :

$$C_t^s = Z_t K_t^{\text{Cd}\alpha} L_t^{\text{Cd}1-\alpha} \quad \left(\lambda_t^{\text{FIRM}^{\text{C}^1}} \right) \quad (2.2)$$

2.2 First order conditions

$$-r_t + \alpha \lambda_t^{\text{FIRM}^{\text{C}^1}} Z_t K_t^{\text{C}^{\text{d}}-1+\alpha} L_t^{\text{C}^{\text{d}}1-\alpha} = 0 \quad (K_t^{\text{C}^{\text{d}}}) \quad (2.3)$$

$$-W_t + \lambda_t^{\text{FIRM}^{\text{C}^1}} Z_t (1 - \alpha) K_t^{\text{C}^{\text{d}}\alpha} L_t^{\text{C}^{\text{d}}-\alpha} = 0 \quad (L_t^{\text{C}^{\text{d}}}) \quad (2.4)$$

$$1 - \lambda_t^{\text{FIRM}^{\text{C}^1}} = 0 \quad (C_t^{\text{s}}) \quad (2.5)$$

2.3 First order conditions after reduction

$$-r_t + \alpha Z_t K_t^{\text{C}^{\text{d}}-1+\alpha} L_t^{\text{C}^{\text{d}}1-\alpha} = 0 \quad (K_t^{\text{C}^{\text{d}}}) \quad (2.6)$$

$$-W_t + Z_t (1 - \alpha) K_t^{\text{C}^{\text{d}}\alpha} L_t^{\text{C}^{\text{d}}-\alpha} = 0 \quad (L_t^{\text{C}^{\text{d}}}) \quad (2.7)$$

3 FIRM I

3.1 Optimisation problem

$$\max_{K_t^{\text{I}^{\text{d}}}, L_t^{\text{I}^{\text{d}}}, I_t^{\text{s}}} \pi_t^{\text{I}} = I_t^{\text{s}} - L_t^{\text{I}^{\text{d}}} W_t - r_t K_t^{\text{I}^{\text{d}}} \quad (3.1)$$

s.t. :

$$I_t^{\text{s}} = Z_t K_t^{\text{I}^{\text{d}}\sigma} L_t^{\text{I}^{\text{d}}1-\alpha} \quad (\lambda_t^{\text{FIRM}^{\text{I}^1}}) \quad (3.2)$$

3.2 First order conditions

$$-r_t + \sigma \lambda_t^{\text{FIRM}^{\text{I}^1}} Z_t K_t^{\text{I}^{\text{d}}-1+\sigma} L_t^{\text{I}^{\text{d}}1-\alpha} = 0 \quad (K_t^{\text{I}^{\text{d}}}) \quad (3.3)$$

$$-W_t + \lambda_t^{\text{FIRM}^{\text{I}^1}} Z_t (1 - \alpha) K_t^{\text{I}^{\text{d}}\sigma} L_t^{\text{I}^{\text{d}}-\alpha} = 0 \quad (L_t^{\text{I}^{\text{d}}}) \quad (3.4)$$

$$1 - \lambda_t^{\text{FIRM}^{\text{I}^1}} = 0 \quad (I_t^{\text{s}}) \quad (3.5)$$

3.3 First order conditions after reduction

$$-r_t + \sigma Z_t K_t^{\text{I}^{\text{d}}-1+\sigma} L_t^{\text{I}^{\text{d}}1-\alpha} = 0 \quad (K_t^{\text{I}^{\text{d}}}) \quad (3.6)$$

$$-W_t + Z_t (1 - \alpha) K_t^{\text{I}^{\text{d}}\sigma} L_t^{\text{I}^{\text{d}}-\alpha} = 0 \quad (L_t^{\text{I}^{\text{d}}}) \quad (3.7)$$

4 EQUILIBRIUM

4.1 Identities

$$K_t^{\text{I}^d} + K_t^{\text{C}^d} = K_{t-1}^{\text{s}} \quad (4.1)$$

$$L_t^{\text{I}^d} + L_t^{\text{C}^d} = L_t^{\text{s}} \quad (4.2)$$

$$C_t = C_t^{\text{s}} \quad (4.3)$$

5 EXOG

5.1 Identities

$$Z_t = e^{\epsilon_t^Z + \phi \log Z_{t-1}} \quad (5.1)$$

6 Equilibrium relationships (after reduction)

$$-r_t + \alpha Z_t K_t^{\text{C}^d} K_t^{\text{C}^d}^{-1+\alpha} L_t^{\text{C}^d} L_t^{\text{C}^d}^{1-\alpha} = 0 \quad (6.1)$$

$$-r_t + \sigma Z_t \left(K_{t-1}^{\text{s}} - K_t^{\text{C}^d} \right)^{-1+\sigma} \left(L_t^{\text{s}} - L_t^{\text{C}^d} \right)^{1-\alpha} = 0 \quad (6.2)$$

$$-C_t + Z_t K_t^{\text{C}^d} L_t^{\text{C}^d} L_t^{\text{C}^d}^{1-\alpha} = 0 \quad (6.3)$$

$$-I_t^{\text{s}} + Z_t \left(K_{t-1}^{\text{s}} - K_t^{\text{C}^d} \right)^{\sigma} \left(L_t^{\text{s}} - L_t^{\text{C}^d} \right)^{1-\alpha} = 0 \quad (6.4)$$

$$-W_t + Z_t (1 - \alpha) K_t^{\text{C}^d} L_t^{\text{C}^d} L_t^{\text{C}^d}^{-\alpha} = 0 \quad (6.5)$$

$$-W_t + Z_t (1 - \alpha) \left(K_{t-1}^{\text{s}} - K_t^{\text{C}^d} \right)^{\sigma} \left(L_t^{\text{s}} - L_t^{\text{C}^d} \right)^{-\alpha} = 0 \quad (6.6)$$

$$-Z_t + e^{\epsilon_t^Z + \phi \log Z_{t-1}} = 0 \quad (6.7)$$

$$\beta \left(\mu \text{E}_t \left[r_{t+1} C_{t+1}^{-1+\mu} (1 - L_{t+1}^{\text{s}})^{1-\mu} \left(C_{t+1}^{\mu} (1 - L_{t+1}^{\text{s}})^{1-\mu} \right)^{-\eta} \right] + \mu (1 - \delta) \text{E}_t \left[p_{t+1} C_{t+1}^{-1+\mu} (1 - L_{t+1}^{\text{s}})^{1-\mu} \left(C_{t+1}^{\mu} (1 - L_{t+1}^{\text{s}})^{1-\mu} \right)^{-\eta} \right] \right) - \mu p_t C_t^{-1+\mu} (1 - L_t^{\text{s}})^{1-\mu} \left(C_t^{\mu} (1 - L_t^{\text{s}})^{1-\mu} \right)^{-\eta} \quad (6.8)$$

$$(-1 + \mu) C_t^\mu (1 - L_t^s)^{-\mu} \left(C_t^\mu (1 - L_t^s)^{1-\mu} \right)^{-\eta} + \mu W_t C_t^{-1+\mu} (1 - L_t^s)^{1-\mu} \left(C_t^\mu (1 - L_t^s)^{1-\mu} \right)^{-\eta} = 0 \quad (6.9)$$

$$C_t - Y_t + p_t I_t = 0 \quad (6.10)$$

$$I_t - K_t^s + K_{t-1}^s (1 - \delta) = 0 \quad (6.11)$$

$$U_t - \beta E_t [U_{t+1}] - (1 - \eta)^{-1} \left(C_t^\mu (1 - L_t^s)^{1-\mu} \right)^{1-\eta} = 0 \quad (6.12)$$

$$I_t^s + K_{t-1}^s r_t - p_t I_t - r_t K_t^{C^d} - r_t \left(K_{t-1}^s - K_t^{C^d} \right) + L_t^s W_t - L_t^{C^d} W_t - W_t \left(L_t^s - L_t^{C^d} \right) = 0 \quad (6.13)$$

7 Steady state relationships (after reduction)

$$-r_{ss} + \alpha Z_{ss} K_{ss}^{C^d - 1 + \alpha} L_{ss}^{C^d 1 - \alpha} = 0 \quad (7.1)$$

$$-r_{ss} + \sigma Z_{ss} \left(K_{ss}^s - K_{ss}^{C^d} \right)^{-1 + \sigma} \left(L_{ss}^s - L_{ss}^{C^d} \right)^{1 - \alpha} = 0 \quad (7.2)$$

$$-C_{ss} + Z_{ss} K_{ss}^{C^d \alpha} L_{ss}^{C^d 1 - \alpha} = 0 \quad (7.3)$$

$$-I_{ss}^s + Z_{ss} \left(K_{ss}^s - K_{ss}^{C^d} \right)^\sigma \left(L_{ss}^s - L_{ss}^{C^d} \right)^{1 - \alpha} = 0 \quad (7.4)$$

$$-W_{ss} + Z_{ss} (1 - \alpha) K_{ss}^{C^d \alpha} L_{ss}^{C^d - \alpha} = 0 \quad (7.5)$$

$$-W_{ss} + Z_{ss} (1 - \alpha) \left(K_{ss}^s - K_{ss}^{C^d} \right)^\sigma \left(L_{ss}^s - L_{ss}^{C^d} \right)^{-\alpha} = 0 \quad (7.6)$$

$$-Z_{ss} + e^{\phi \log Z_{ss}} = 0 \quad (7.7)$$

$$\beta \left(\mu r_{ss} C_{ss}^{-1+\mu} (1 - L_{ss}^s)^{1-\mu} \left(C_{ss}^\mu (1 - L_{ss}^s)^{1-\mu} \right)^{-\eta} + \mu p_{ss} (1 - \delta) C_{ss}^{-1+\mu} (1 - L_{ss}^s)^{1-\mu} \left(C_{ss}^\mu (1 - L_{ss}^s)^{1-\mu} \right)^{-\eta} \right) - \mu p_{ss} C_{ss}^{-1+\mu} (1 - L_{ss}^s)^{1-\mu} \left(C_{ss}^\mu (1 - L_{ss}^s)^{1-\mu} \right)^{-\eta} = 0 \quad (7.8)$$

$$(-1 + \mu) C_{ss}^\mu (1 - L_{ss}^s)^{-\mu} \left(C_{ss}^\mu (1 - L_{ss}^s)^{1-\mu} \right)^{-\eta} + \mu W_{ss} C_{ss}^{-1+\mu} (1 - L_{ss}^s)^{1-\mu} \left(C_{ss}^\mu (1 - L_{ss}^s)^{1-\mu} \right)^{-\eta} = 0 \quad (7.9)$$

$$C_{ss} - Y_{ss} + p_{ss}I_{ss} = 0 \quad (7.10)$$

$$I_{ss} - K_{ss}^s + K_{ss}^s(1 - \delta) = 0 \quad (7.11)$$

$$U_{ss} - \beta U_{ss} - (1 - \eta)^{-1} \left(C_{ss}^\mu (1 - L_{ss}^s)^{1-\mu} \right)^{1-\eta} = 0 \quad (7.12)$$

$$I_{ss}^s - p_{ss}I_{ss} + r_{ss}K_{ss}^s - r_{ss}K_{ss}^{C^d} - r_{ss} \left(K_{ss}^s - K_{ss}^{C^d} \right) + L_{ss}^s W_{ss} - L_{ss}^{C^d} W_{ss} - W_{ss} \left(L_{ss}^s - L_{ss}^{C^d} \right) = 0 \quad (7.13)$$

8 Parameter settings

$$\alpha = 0.2 \quad (8.1)$$

$$\beta = 0.99 \quad (8.2)$$

$$\delta = 0.025 \quad (8.3)$$

$$\eta = 2 \quad (8.4)$$

$$\mu = 0.3 \quad (8.5)$$

$$\phi = 0.95 \quad (8.6)$$

$$\sigma = 0.4 \quad (8.7)$$

9 Steady-state values

	Steady-state value
p	1.5318
r	0.0538
C	0.3374
I	0.0439
I^s	0.0672
K^s	1.7551
K^{C^d}	1.2551
L^s	0.2914
L^{C^d}	0.243
U	-176.3002
W	1.111
Y	0.4046
Z	1

10 The solution of the 1st order perturbation

Matrix P

$$\begin{matrix} K_t^s & Z_{t-1} \\ Z_t \end{matrix} \begin{pmatrix} 0.9522 & -0.0054 \\ 0 & 0.95 \end{pmatrix}$$

Matrix Q

$$\begin{matrix} \epsilon^Z \\ Z \end{matrix} \begin{pmatrix} -0.0056 \\ 1 \end{pmatrix}$$

Matrix R

$$\begin{matrix} p_t \\ r_t \\ C_t \\ I_t \\ I_t^s \\ K_t^{C^d} \\ L_t^s \\ L_t^{C^d} \\ U_t \\ W_t \\ Y_t \end{matrix} \begin{pmatrix} K_{t-1}^s & Z_{t-1} \\ -0.1506 & 1.1645 \\ -1.0646 & 0.95 \\ 0.3338 & 0.95 \\ -0.9139 & -0.2145 \\ -1.0646 & 0.95 \\ 1.3984 & 0 \\ -0.1646 & 0 \\ 0.0677 & 0 \\ 0.0257 & 0.0456 \\ 0.2661 & 0.95 \\ 0.1015 & 0.95 \end{pmatrix}$$

Matrix S

$$\begin{matrix} p \\ r \\ C \\ I \\ I^s \\ K^{C^d} \\ L^s \\ L^{C^d} \\ U \\ W \\ Y \end{matrix} \begin{matrix} \epsilon^Z \\ \left(\begin{array}{c} 1.2258 \\ 1 \\ 1 \\ -0.2258 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0.048 \\ 1 \\ 1 \end{array} \right) \end{matrix}$$

11 Model statistics

11.1 Basic statistics

	Steady-state value	Std. dev.	Variance	Loglin
p	1.5318	0.113	0.0128	Y
r	0.0538	0.0922	0.0085	Y
C	0.3374	0.0922	0.0085	Y
I	0.0439	0.0208	0.0004	Y
I^s	0.0672	0.0922	0.0085	Y
K^s	1.7551	0.0018	0	Y
L^s	0.2914	0	0	Y
U	-176.3002	0.0044	0	Y
W	1.111	0.0922	0.0085	Y
Y	0.4046	0.0922	0.0085	Y

11.2 Correlation matrix

	p	r	C	I	I^s	K^s	U	W	Y
p	1	1	1	-0.997	1	-0.309	1	1	1
r		1	1	-0.995	1	-0.327	0.999	1	1
C			1	-0.997	1	-0.3	1	1	1
I				1	-0.995	0.229	-0.998	-0.997	-0.997
I^s					1	-0.327	0.999	1	1
K^s						1	-0.296	-0.301	-0.305
U							1	1	1
W								1	1
Y									1

11.3 Cross correlations with the reference variable (Y)

	$\sigma[\cdot]$ rel. to $\sigma[Y]$	Y_{t-5}	Y_{t-4}	Y_{t-3}	Y_{t-2}	Y_{t-1}	Y_t	Y_{t+1}	Y_{t+2}	Y_{t+3}	Y_{t+4}	Y_{t+5}
p_t	1.226	-0.019	0.107	0.269	0.469	0.712	1	0.714	0.473	0.273	0.112	-0.014
r_t	1.001	-0.028	0.098	0.26	0.463	0.708	1	0.72	0.482	0.285	0.125	-0.002
C_t	1	-0.014	0.112	0.273	0.472	0.714	1	0.712	0.469	0.268	0.107	-0.02
I_t	0.226	-0.023	-0.148	-0.305	-0.497	-0.727	-0.997	-0.688	-0.432	-0.223	-0.058	0.068
I_t^s	1.001	-0.028	0.098	0.26	0.463	0.708	1	0.72	0.482	0.285	0.125	-0.002
K_t^s	0.02	0.491	0.437	0.339	0.19	-0.022	-0.305	-0.493	-0.603	-0.651	-0.652	-0.616
U_t	0.048	-0.012	0.114	0.275	0.474	0.715	1	0.711	0.467	0.266	0.104	-0.022
W_t	1	-0.015	0.111	0.272	0.472	0.714	1	0.712	0.469	0.269	0.107	-0.019
Y_t	1	-0.017	0.11	0.271	0.471	0.713	1	0.713	0.471	0.271	0.11	-0.017

11.4 Autocorrelations

	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5
p	0.713	0.471	0.271	0.11	-0.016
r	0.715	0.474	0.275	0.113	-0.013
C	0.713	0.47	0.27	0.109	-0.017
I	0.708	0.463	0.261	0.1	-0.025
I^s	0.715	0.474	0.275	0.113	-0.013
K^s	0.959	0.86	0.723	0.565	0.399
U	0.712	0.47	0.269	0.108	0
W	0.713	0.47	0.27	0.109	-0.017
Y	0.713	0.471	0.271	0.11	-0.017

12 Impulse response functions

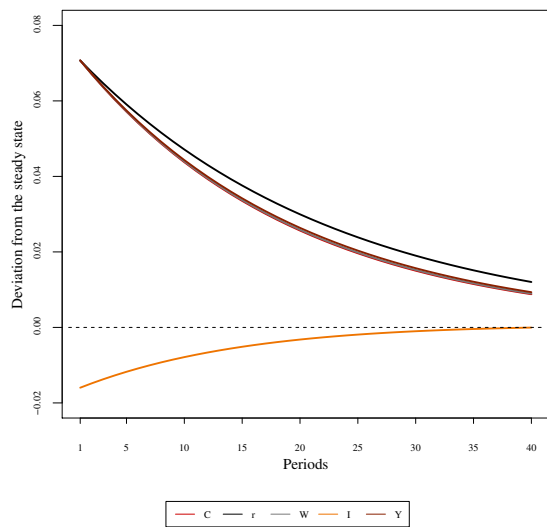


Figure 1: Impulse responses (C, r, W, I, Y) to ϵ^Z shock

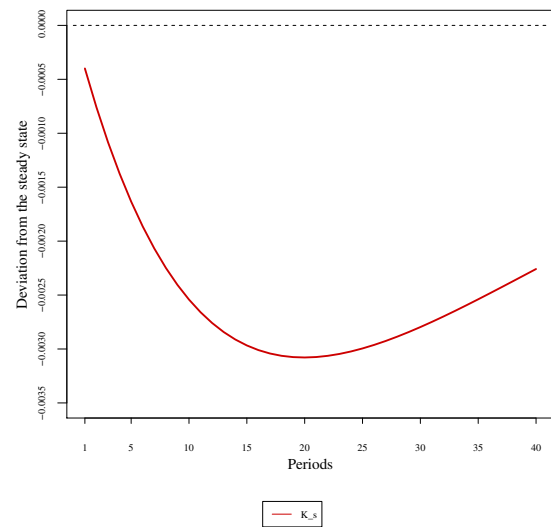


Figure 2: Impulse response (K^s) to ϵ^Z shock