

Index sets

$$COUNTRY = \{F, H\}$$

1 CONSUMER $c \in COUNTRY$

1.1 Optimisation problem

$$\max_{K_t^{(c)}, C_t^{(c)}, H_t^{(c)}, I_t^{(c)}} U_t^{(c)} = \beta E_t \left[U_{t+1}^{(c)} \right] + (1 - \eta)^{-1} \left(C_t^{(c)\mu} (1 - H_t^{(c)})^{1-\mu} \right)^{1-\eta} \quad (1.1)$$

s.t. :

$$C_t^{(c)} + I_t^{(c)} + T_t^{(c)} = \pi_t^{(c)} + TR_t^{(c)} + K_{t-1}^{(c)} r_t^{(c)} + H_t^{(c)} W_t^{(c)} - \psi^{(c)} K_{t-1}^{(c)} \left(-\delta^{(c)} + K_{t-1}^{(c)-1} I_t^{(c)} \right)^2 \quad \left(\lambda_t^{c(c)} \right) \quad (1.2)$$

$$K_t^{(c)} = I_t^{(c)} + K_{t-1}^{(c)} (1 - \delta^{(c)}) \quad \left(\lambda^{CONSUMER^2_t(c)} \right) \quad (1.3)$$

1.2 First order conditions

$$-\lambda^{CONSUMER^2_t(c)} + \beta \left((1 - \delta^{(c)}) E_t \left[\lambda^{CONSUMER^2_{t+1}(c)} \right] + E_t \left[\lambda_{t+1}^{c(c)} \left(r_{t+1}^{(c)} - \psi^{(c)} \left(-\delta^{(c)} + K_t^{(c)-1} I_{t+1}^{(c)} \right)^2 + 2\psi^{(c)} K_t^{(c)-1} I_{t+1}^{(c)} \left(-\delta^{(c)} + K_t^{(c)-1} I_{t+1}^{(c)} \right) \right] \right) \right) = 0 \quad \left(K_t^{(c)} \right) \quad (1.4)$$

$$-\lambda_t^{c(c)} + \mu C_t^{(c)-1+\mu} (1 - H_t^{(c)})^{1-\mu} \left(C_t^{(c)\mu} (1 - H_t^{(c)})^{1-\mu} \right)^{-\eta} = 0 \quad \left(C_t^{(c)} \right) \quad (1.5)$$

$$\lambda_t^{c(c)} W_t^{(c)} + (-1 + \mu) C_t^{(c)\mu} (1 - H_t^{(c)})^{-\mu} \left(C_t^{(c)\mu} (1 - H_t^{(c)})^{1-\mu} \right)^{-\eta} = 0 \quad \left(H_t^{(c)} \right) \quad (1.6)$$

$$\lambda^{CONSUMER^2_t(c)} + \lambda_t^{c(c)} \left(-1 - 2\psi^{(c)} \left(-\delta^{(c)} + K_{t-1}^{(c)-1} I_t^{(c)} \right) \right) = 0 \quad \left(I_t^{(c)} \right) \quad (1.7)$$

2 FIRM $c \in \text{COUNTRY}$

2.1 Optimisation problem

$$\max_{K_t^{d(c)}, H_t^{d(c)}, Y_t^{(c)}, \pi_t^{(c)}} \Pi_t^{(c)} = \pi_t^{(c)} \quad (2.1)$$

s.t. :

$$Y_t^{(c)} = Z_t^{(c)} H_t^{d(c)1-\alpha^{(c)}} K_t^{d(c)\alpha^{(c)}} \left(\lambda^{\text{FIRM}^1(c)} \right) \quad (2.2)$$

$$\pi_t^{(c)} = Y_t^{(c)} - H_t^{d(c)} W_t^{(c)} - r_t^{(c)} K_t^{d(c)} \left(\lambda^{\text{FIRM}^2(c)} \right) \quad (2.3)$$

2.2 First order conditions

$$-\lambda^{\text{FIRM}^2(c)} r_t^{(c)} + \alpha^{(c)} \lambda^{\text{FIRM}^1(c)} Z_t^{(c)} H_t^{d(c)1-\alpha^{(c)}} K_t^{d(c)-1+\alpha^{(c)}} = 0 \quad \left(K_t^{d(c)} \right) \quad (2.4)$$

$$-\lambda^{\text{FIRM}^2(c)} W_t^{(c)} + \lambda^{\text{FIRM}^1(c)} Z_t^{(c)} \left(1 - \alpha^{(c)} \right) H_t^{d(c)-\alpha^{(c)}} K_t^{d(c)\alpha^{(c)}} = 0 \quad \left(H_t^{d(c)} \right) \quad (2.5)$$

$$-\lambda^{\text{FIRM}^1(c)} + \lambda^{\text{FIRM}^2(c)} = 0 \quad \left(Y_t^{(c)} \right) \quad (2.6)$$

$$1 - \lambda^{\text{FIRM}^2(c)} = 0 \quad \left(\pi_t^{(c)} \right) \quad (2.7)$$

2.3 First order conditions after reduction

$$-r_t^{(c)} + \alpha^{(c)} Z_t^{(c)} H_t^{d(c)1-\alpha^{(c)}} K_t^{d(c)-1+\alpha^{(c)}} = 0 \quad \left(K_t^{d(c)} \right) \quad (2.8)$$

$$-W_t^{(c)} + Z_t^{(c)} \left(1 - \alpha^{(c)} \right) H_t^{d(c)-\alpha^{(c)}} K_t^{d(c)\alpha^{(c)}} = 0 \quad \left(H_t^{d(c)} \right) \quad (2.9)$$

3 EQUILIBRIUM

3.1 Identities

$$\sum_{c \in \text{COUNTRY}} TR_t^{(c)} = 0 \quad (3.1)$$

$$c \in \text{COUNTRY} : K_t^{d(c)} = K_{t-1}^{(c)} \quad (3.2)$$

$$c \in \text{COUNTRY} : H_t^{d(c)} = H_t^{(c)} \quad (3.3)$$

$$c \in \text{COUNTRY} : T_t^{(c)} = G_t^{d(c)} \quad (3.4)$$

$$\lambda_t^{c(H)} = \lambda_t^{c(F)} \quad (3.5)$$

4 EXOG

4.1 Identities

$$c \in \text{COUNTRY} : G_t^{d(c)} = \epsilon_t^{G(c)} + \phi^{G(c)} G_{t-1}^{d(c)} \quad (4.1)$$

$$c \in \text{COUNTRY} : Z_t^{(c)} = e^{\epsilon_t^{Z(c)} + \phi^{Z(c)} \log Z_{t-1}^{(c)}} \quad (4.2)$$

5 Equilibrium relationships (before expansion and reduction)

$$- \sum_{c \in \text{COUNTRY}} TR_t^{(c)} = 0 \quad (5.1)$$

$$\lambda_t^{c(F)} - \lambda_t^{c(H)} = 0 \quad (5.2)$$

$$c \in \text{COUNTRY} : K_{t-1}^{(c)} - K_t^{d(c)} = 0 \quad (5.3)$$

$$c \in \text{COUNTRY} : -\lambda^{\text{CONSUMER}^2(c)}_t + \beta \left((1 - \delta^{(c)}) E_t \left[\lambda^{\text{CONSUMER}^2(c)}_{t+1} \right] + E_t \left[\lambda_{t+1}^{c(c)} \left(r_{t+1}^{(c)} - \psi^{(c)} \left(-\delta^{(c)} + K_t^{(c)-1} I_{t+1}^{(c)} \right)^2 + 2\psi^{(c)} K_t^{(c)-1} I_{t+1}^{(c)} \left(-\delta^{(c)} + K_t^{(c)-1} I_{t+1}^{(c)} \right) \right] \right) \right) = 0 \quad (5.4)$$

$$c \in \text{COUNTRY} : \lambda^{\text{CONSUMER}^2(c)}_t + \lambda_t^{c(c)} \left(-1 - 2\psi^{(c)} \left(-\delta^{(c)} + K_{t-1}^{(c)-1} I_t^{(c)} \right) \right) = 0 \quad (5.5)$$

$$c \in \text{COUNTRY} : -\lambda_t^{c(c)} + \mu C_t^{(c)-1+\mu} \left(1 - H_t^{(c)} \right)^{1-\mu} \left(C_t^{(c)\mu} \left(1 - H_t^{(c)} \right)^{1-\mu} \right)^{-\eta} = 0 \quad (5.6)$$

$$c \in \text{COUNTRY} : -\pi_t^{(c)} + \Pi_t^{(c)} = 0 \quad (5.7)$$

$$c \in \text{COUNTRY} : \quad -r_t^{(c)} + \alpha^{(c)} Z_t^{(c)} H_t^{d(c)1-\alpha^{(c)}} K_t^{d(c)-1+\alpha^{(c)}} = 0 \quad (5.8)$$

$$c \in \text{COUNTRY} : \quad G_t^{d(c)} - T_t^{(c)} = 0 \quad (5.9)$$

$$c \in \text{COUNTRY} : \quad H_t^{(c)} - H_t^{d(c)} = 0 \quad (5.10)$$

$$c \in \text{COUNTRY} : \quad -W_t^{(c)} + Z_t^{(c)} \left(1 - \alpha^{(c)}\right) H_t^{d(c)-\alpha^{(c)}} K_t^{d(c)\alpha^{(c)}} = 0 \quad (5.11)$$

$$c \in \text{COUNTRY} : \quad -Y_t^{(c)} + Z_t^{(c)} H_t^{d(c)1-\alpha^{(c)}} K_t^{d(c)\alpha^{(c)}} = 0 \quad (5.12)$$

$$c \in \text{COUNTRY} : \quad -Z_t^{(c)} + e^{e^{Z_t^{(c)}} + \phi^{Z_t^{(c)}} \log Z_{t-1}^{(c)}} = 0 \quad (5.13)$$

$$c \in \text{COUNTRY} : \quad \lambda_t^{c(c)} W_t^{(c)} + (-1 + \mu) C_t^{(c)\mu} \left(1 - H_t^{(c)}\right)^{-\mu} \left(C_t^{(c)\mu} \left(1 - H_t^{(c)}\right)^{1-\mu}\right)^{-\eta} = 0 \quad (5.14)$$

$$c \in \text{COUNTRY} : \quad \epsilon_t^{G(c)} - G_t^{d(c)} + \phi^{G(c)} G_{t-1}^{d(c)} = 0 \quad (5.15)$$

$$c \in \text{COUNTRY} : \quad I_t^{(c)} - K_t^{(c)} + K_{t-1}^{(c)} \left(1 - \delta^{(c)}\right) = 0 \quad (5.16)$$

$$c \in \text{COUNTRY} : \quad U_t^{(c)} - \beta E_t \left[U_{t+1}^{(c)}\right] - (1 - \eta)^{-1} \left(C_t^{(c)\mu} \left(1 - H_t^{(c)}\right)^{1-\mu}\right)^{1-\eta} = 0 \quad (5.17)$$

$$c \in \text{COUNTRY} : \quad -\pi_t^{(c)} + Y_t^{(c)} - r_t^{(c)} K_t^{d(c)} - H_t^{d(c)} W_t^{(c)} = 0 \quad (5.18)$$

$$c \in \text{COUNTRY} : \quad \pi_t^{(c)} - C_t^{(c)} - I_t^{(c)} - T_t^{(c)} + TR_t^{(c)} + K_{t-1}^{(c)} r_t^{(c)} + H_t^{(c)} W_t^{(c)} - \psi^{(c)} K_{t-1}^{(c)} \left(-\delta^{(c)} + K_{t-1}^{(c)-1} I_t^{(c)}\right)^2 = 0 \quad (5.19)$$

6 Equilibrium relationships (after expansion and reduction)

$$-\lambda_t^{c\langle F \rangle} + \mu C_t^{\langle F \rangle - 1 + \mu} \left(1 - H_t^{\langle F \rangle}\right)^{1 - \mu} \left(C_t^{\langle F \rangle \mu} \left(1 - H_t^{\langle F \rangle}\right)^{1 - \mu}\right)^{-\eta} = 0 \quad (6.1)$$

$$\lambda_t^{c\langle F \rangle} - \lambda_t^{c\langle H \rangle} = 0 \quad (6.2)$$

$$-\lambda_t^{c\langle H \rangle} + \mu C_t^{\langle H \rangle - 1 + \mu} \left(1 - H_t^{\langle H \rangle}\right)^{1 - \mu} \left(C_t^{\langle H \rangle \mu} \left(1 - H_t^{\langle H \rangle}\right)^{1 - \mu}\right)^{-\eta} = 0 \quad (6.3)$$

$$-r_t^{\langle F \rangle} + \alpha^{\langle F \rangle} Z_t^{\langle F \rangle} K_{t-1}^{\langle F \rangle - 1 + \alpha^{\langle F \rangle}} H_t^{\langle F \rangle 1 - \alpha^{\langle F \rangle}} = 0 \quad (6.4)$$

$$-r_t^{\langle H \rangle} + \alpha^{\langle H \rangle} Z_t^{\langle H \rangle} K_{t-1}^{\langle H \rangle - 1 + \alpha^{\langle H \rangle}} H_t^{\langle H \rangle 1 - \alpha^{\langle H \rangle}} = 0 \quad (6.5)$$

$$-W_t^{\langle F \rangle} + Z_t^{\langle F \rangle} \left(1 - \alpha^{\langle F \rangle}\right) K_{t-1}^{\langle F \rangle \alpha^{\langle F \rangle}} H_t^{\langle F \rangle - \alpha^{\langle F \rangle}} = 0 \quad (6.6)$$

$$-W_t^{\langle H \rangle} + Z_t^{\langle H \rangle} \left(1 - \alpha^{\langle H \rangle}\right) K_{t-1}^{\langle H \rangle \alpha^{\langle H \rangle}} H_t^{\langle H \rangle - \alpha^{\langle H \rangle}} = 0 \quad (6.7)$$

$$-Y_t^{\langle F \rangle} + Z_t^{\langle F \rangle} K_{t-1}^{\langle F \rangle \alpha^{\langle F \rangle}} H_t^{\langle F \rangle 1 - \alpha^{\langle F \rangle}} = 0 \quad (6.8)$$

$$-Y_t^{\langle H \rangle} + Z_t^{\langle H \rangle} K_{t-1}^{\langle H \rangle \alpha^{\langle H \rangle}} H_t^{\langle H \rangle 1 - \alpha^{\langle H \rangle}} = 0 \quad (6.9)$$

$$-Z_t^{\langle F \rangle} + e^{\epsilon_t^{Z\langle F \rangle} + \phi_t^{Z\langle F \rangle} \log Z_{t-1}^{\langle F \rangle}} = 0 \quad (6.10)$$

$$-Z_t^{\langle H \rangle} + e^{\epsilon_t^{Z\langle H \rangle} + \phi_t^{Z\langle H \rangle} \log Z_{t-1}^{\langle H \rangle}} = 0 \quad (6.11)$$

$$\beta \left(- \left(1 - \delta^{\langle F \rangle}\right) \mathbf{E}_t \left[\lambda_{t+1}^{c\langle F \rangle} \left(-1 - 2\psi^{\langle F \rangle} \left(-\delta^{\langle F \rangle} + K_t^{\langle F \rangle - 1} I_{t+1}^{\langle F \rangle} \right) \right) \right] + \mathbf{E}_t \left[\lambda_{t+1}^{c\langle F \rangle} \left(r_{t+1}^{\langle F \rangle} - \psi^{\langle F \rangle} \left(-\delta^{\langle F \rangle} + K_t^{\langle F \rangle - 1} I_{t+1}^{\langle F \rangle} \right)^2 + 2\psi^{\langle F \rangle} K_t^{\langle F \rangle - 1} I_{t+1}^{\langle F \rangle} \left(-\delta^{\langle F \rangle} + K_t^{\langle F \rangle - 1} I_{t+1}^{\langle F \rangle} \right) \right) \right] \right) + \lambda_t^{c\langle F \rangle} \left(-1 - 2\psi^{\langle F \rangle} \left(-\delta^{\langle F \rangle} + K_t^{\langle F \rangle - 1} I_{t+1}^{\langle F \rangle} \right) \right) \quad (6.12)$$

$$\beta \left(- \left(1 - \delta^{\langle H \rangle}\right) \mathbf{E}_t \left[\lambda_{t+1}^{c\langle H \rangle} \left(-1 - 2\psi^{\langle H \rangle} \left(-\delta^{\langle H \rangle} + K_t^{\langle H \rangle - 1} I_{t+1}^{\langle H \rangle} \right) \right) \right] + \mathbf{E}_t \left[\lambda_{t+1}^{c\langle H \rangle} \left(r_{t+1}^{\langle H \rangle} - \psi^{\langle H \rangle} \left(-\delta^{\langle H \rangle} + K_t^{\langle H \rangle - 1} I_{t+1}^{\langle H \rangle} \right)^2 + 2\psi^{\langle H \rangle} K_t^{\langle H \rangle - 1} I_{t+1}^{\langle H \rangle} \left(-\delta^{\langle H \rangle} + K_t^{\langle H \rangle - 1} I_{t+1}^{\langle H \rangle} \right) \right) \right] \right) + \lambda_t^{c\langle H \rangle} \left(-1 - 2\psi^{\langle H \rangle} \left(-\delta^{\langle H \rangle} + K_t^{\langle H \rangle - 1} I_{t+1}^{\langle H \rangle} \right) \right) \quad (6.13)$$

$$\lambda_t^{c\langle F \rangle} W_t^{(F)} + (-1 + \mu) C_t^{(F)\mu} (1 - H_t^{(F)})^{-\mu} \left(C_t^{(F)\mu} (1 - H_t^{(F)})^{1-\mu} \right)^{-\eta} = 0 \quad (6.14)$$

$$\lambda_t^{c\langle H \rangle} W_t^{(H)} + (-1 + \mu) C_t^{(H)\mu} (1 - H_t^{(H)})^{-\mu} \left(C_t^{(H)\mu} (1 - H_t^{(H)})^{1-\mu} \right)^{-\eta} = 0 \quad (6.15)$$

$$\epsilon_t^{G\langle F \rangle} - G_t^{d\langle F \rangle} + \phi^{G\langle F \rangle} G_{t-1}^{d\langle F \rangle} = 0 \quad (6.16)$$

$$\epsilon_t^{G\langle H \rangle} - G_t^{d\langle H \rangle} + \phi^{G\langle H \rangle} G_{t-1}^{d\langle H \rangle} = 0 \quad (6.17)$$

$$I_t^{(F)} - K_t^{(F)} + K_{t-1}^{(F)} (1 - \delta^{(F)}) = 0 \quad (6.18)$$

$$I_t^{(H)} - K_t^{(H)} + K_{t-1}^{(H)} (1 - \delta^{(H)}) = 0 \quad (6.19)$$

$$U_t^{(F)} - \beta E_t [U_{t+1}^{(F)}] - (1 - \eta)^{-1} \left(C_t^{(F)\mu} (1 - H_t^{(F)})^{1-\mu} \right)^{1-\eta} = 0 \quad (6.20)$$

$$U_t^{(H)} - \beta E_t [U_{t+1}^{(H)}] - (1 - \eta)^{-1} \left(C_t^{(H)\mu} (1 - H_t^{(H)})^{1-\mu} \right)^{1-\eta} = 0 \quad (6.21)$$

$$-C_t^{(F)} - G_t^{d\langle F \rangle} - I_t^{(F)} - TR_t^{(H)} + Y_t^{(F)} - \psi^{(F)} K_{t-1}^{(F)} \left(-\delta^{(F)} + K_{t-1}^{(F)-1} I_t^{(F)} \right)^2 = 0 \quad (6.22)$$

$$-C_t^{(H)} - G_t^{d\langle H \rangle} - I_t^{(H)} + TR_t^{(H)} + Y_t^{(H)} - \psi^{(H)} K_{t-1}^{(H)} \left(-\delta^{(H)} + K_{t-1}^{(H)-1} I_t^{(H)} \right)^2 = 0 \quad (6.23)$$

7 Steady state relationships (before expansion and reduction)

$$- \sum_{c \in COUNTRY} TR_{ss}^{(c)} = 0 \quad (7.1)$$

$$\lambda_{ss}^{c\langle F \rangle} - \lambda_{ss}^{c\langle H \rangle} = 0 \quad (7.2)$$

$$c \in COUNTRY: \quad K_{ss}^{(c)} - K_{ss}^{d\langle c \rangle} = 0 \quad (7.3)$$

$$c \in COUNTRY: \quad -\lambda_{ss}^{CONSUMER^2\langle c \rangle} + \beta \left(\lambda_{ss}^{CONSUMER^2\langle c \rangle} (1 - \delta^{(c)}) + \lambda_{ss}^{c\langle c \rangle} \left(r_{ss}^{(c)} - \psi^{(c)} \left(-\delta^{(c)} + I_{ss}^{(c)} K_{ss}^{(c)-1} \right)^2 + 2\psi^{(c)} I_{ss}^{(c)} K_{ss}^{(c)-1} \left(-\delta^{(c)} + I_{ss}^{(c)} K_{ss}^{(c)-1} \right) \right) \right) = 0 \quad (7.4)$$

$$c \in \text{COUNTRY} : \quad \lambda^{\text{CONSUMER}^2(c)}_{ss} + \lambda_{ss}^{c(c)} \left(-1 - 2\psi^{(c)} \left(-\delta^{(c)} + I_{ss}^{(c)} K_{ss}^{(c)-1} \right) \right) = 0 \quad (7.5)$$

$$c \in \text{COUNTRY} : \quad -\lambda_{ss}^{c(c)} + \mu C_{ss}^{(c)-1+\mu} \left(1 - H_{ss}^{(c)} \right)^{1-\mu} \left(C_{ss}^{(c)\mu} \left(1 - H_{ss}^{(c)} \right)^{1-\mu} \right)^{-\eta} = 0 \quad (7.6)$$

$$c \in \text{COUNTRY} : \quad -\pi_{ss}^{(c)} + \Pi_{ss}^{(c)} = 0 \quad (7.7)$$

$$c \in \text{COUNTRY} : \quad -r_{ss}^{(c)} + \alpha^{(c)} Z_{ss}^{(c)} H_{ss}^{d(c)1-\alpha^{(c)}} K_{ss}^{d(c)-1+\alpha^{(c)}} = 0 \quad (7.8)$$

$$c \in \text{COUNTRY} : \quad G_{ss}^{d(c)} - T_{ss}^{(c)} = 0 \quad (7.9)$$

$$c \in \text{COUNTRY} : \quad H_{ss}^{(c)} - H_{ss}^{d(c)} = 0 \quad (7.10)$$

$$c \in \text{COUNTRY} : \quad -W_{ss}^{(c)} + Z_{ss}^{(c)} \left(1 - \alpha^{(c)} \right) H_{ss}^{d(c)-\alpha^{(c)}} K_{ss}^{d(c)\alpha^{(c)}} = 0 \quad (7.11)$$

$$c \in \text{COUNTRY} : \quad -Y_{ss}^{(c)} + Z_{ss}^{(c)} H_{ss}^{d(c)1-\alpha^{(c)}} K_{ss}^{d(c)\alpha^{(c)}} = 0 \quad (7.12)$$

$$c \in \text{COUNTRY} : \quad -Z_{ss}^{(c)} + e^{\epsilon_{ss}^{Z(c)} + \phi^{Z(c)} \log Z_{ss}^{(c)}} = 0 \quad (7.13)$$

$$c \in \text{COUNTRY} : \quad \lambda_{ss}^{c(c)} W_{ss}^{(c)} + (-1 + \mu) C_{ss}^{(c)\mu} \left(1 - H_{ss}^{(c)} \right)^{-\mu} \left(C_{ss}^{(c)\mu} \left(1 - H_{ss}^{(c)} \right)^{1-\mu} \right)^{-\eta} = 0 \quad (7.14)$$

$$c \in \text{COUNTRY} : \quad \epsilon_{ss}^{G(c)} - G_{ss}^{d(c)} + \phi^{G(c)} G_{ss}^{d(c)} = 0 \quad (7.15)$$

$$c \in \text{COUNTRY} : \quad I_{ss}^{(c)} - K_{ss}^{(c)} + K_{ss}^{(c)} \left(1 - \delta^{(c)} \right) = 0 \quad (7.16)$$

$$c \in \text{COUNTRY} : \quad U_{ss}^{(c)} - \beta U_{ss}^{(c)} - (1 - \eta)^{-1} \left(C_{ss}^{(c)\mu} \left(1 - H_{ss}^{(c)} \right)^{1-\mu} \right)^{1-\eta} = 0 \quad (7.17)$$

$$c \in \text{COUNTRY} : \quad -\pi_{ss}^{(c)} + Y_{ss}^{(c)} - r_{ss}^{(c)} K_{ss}^{d(c)} - H_{ss}^{d(c)} W_{ss}^{(c)} = 0 \quad (7.18)$$

$$c \in \text{COUNTRY} : \quad \pi_{ss}^{(c)} - C_{ss}^{(c)} - I_{ss}^{(c)} - T_{ss}^{(c)} + TR_{ss}^{(c)} + r_{ss}^{(c)} K_{ss}^{(c)} + H_{ss}^{(c)} W_{ss}^{(c)} - \psi^{(c)} K_{ss}^{(c)} \left(-\delta^{(c)} + I_{ss}^{(c)} K_{ss}^{(c)-1} \right)^2 = 0 \quad (7.19)$$

8 Steady state relationships (after expansion and reduction)

$$-\lambda_{ss}^{c\langle F \rangle} + \mu C_{ss}^{\langle F \rangle -1+\mu} \left(1 - H_{ss}^{\langle F \rangle}\right)^{1-\mu} \left(C_{ss}^{\langle F \rangle \mu} \left(1 - H_{ss}^{\langle F \rangle}\right)^{1-\mu}\right)^{-\eta} = 0 \quad (8.1)$$

$$\lambda_{ss}^{c\langle F \rangle} - \lambda_{ss}^{c\langle H \rangle} = 0 \quad (8.2)$$

$$-\lambda_{ss}^{c\langle H \rangle} + \mu C_{ss}^{\langle H \rangle -1+\mu} \left(1 - H_{ss}^{\langle H \rangle}\right)^{1-\mu} \left(C_{ss}^{\langle H \rangle \mu} \left(1 - H_{ss}^{\langle H \rangle}\right)^{1-\mu}\right)^{-\eta} = 0 \quad (8.3)$$

$$-r_{ss}^{\langle F \rangle} + \alpha^{\langle F \rangle} Z_{ss}^{\langle F \rangle} H_{ss}^{\langle F \rangle 1-\alpha^{\langle F \rangle}} K_{ss}^{\langle F \rangle -1+\alpha^{\langle F \rangle}} = 0 \quad (8.4)$$

$$-r_{ss}^{\langle H \rangle} + \alpha^{\langle H \rangle} Z_{ss}^{\langle H \rangle} H_{ss}^{\langle H \rangle 1-\alpha^{\langle H \rangle}} K_{ss}^{\langle H \rangle -1+\alpha^{\langle H \rangle}} = 0 \quad (8.5)$$

$$-W_{ss}^{\langle F \rangle} + Z_{ss}^{\langle F \rangle} \left(1 - \alpha^{\langle F \rangle}\right) H_{ss}^{\langle F \rangle -\alpha^{\langle F \rangle}} K_{ss}^{\langle F \rangle \alpha^{\langle F \rangle}} = 0 \quad (8.6)$$

$$-W_{ss}^{\langle H \rangle} + Z_{ss}^{\langle H \rangle} \left(1 - \alpha^{\langle H \rangle}\right) H_{ss}^{\langle H \rangle -\alpha^{\langle H \rangle}} K_{ss}^{\langle H \rangle \alpha^{\langle H \rangle}} = 0 \quad (8.7)$$

$$-Y_{ss}^{\langle F \rangle} + Z_{ss}^{\langle F \rangle} H_{ss}^{\langle F \rangle 1-\alpha^{\langle F \rangle}} K_{ss}^{\langle F \rangle \alpha^{\langle F \rangle}} = 0 \quad (8.8)$$

$$-Y_{ss}^{\langle H \rangle} + Z_{ss}^{\langle H \rangle} H_{ss}^{\langle H \rangle 1-\alpha^{\langle H \rangle}} K_{ss}^{\langle H \rangle \alpha^{\langle H \rangle}} = 0 \quad (8.9)$$

$$-Z_{ss}^{\langle F \rangle} + e^{\phi^{Z\langle F \rangle} \log Z_{ss}^{\langle F \rangle}} = 0 \quad (8.10)$$

$$-Z_{ss}^{\langle H \rangle} + e^{\phi^{Z\langle H \rangle} \log Z_{ss}^{\langle H \rangle}} = 0 \quad (8.11)$$

$$\beta \left(\lambda_{ss}^{c\langle F \rangle} \left(r_{ss}^{\langle F \rangle} - \psi^{\langle F \rangle} \left(-\delta^{\langle F \rangle} + I_{ss}^{\langle F \rangle} K_{ss}^{\langle F \rangle -1} \right)^2 + 2\psi^{\langle F \rangle} I_{ss}^{\langle F \rangle} K_{ss}^{\langle F \rangle -1} \left(-\delta^{\langle F \rangle} + I_{ss}^{\langle F \rangle} K_{ss}^{\langle F \rangle -1} \right) \right) - \lambda_{ss}^{c\langle F \rangle} \left(-1 - 2\psi^{\langle F \rangle} \left(-\delta^{\langle F \rangle} + I_{ss}^{\langle F \rangle} K_{ss}^{\langle F \rangle -1} \right) \right) \left(1 - \delta^{\langle F \rangle} \right) \right) + \lambda_{ss}^{c\langle F \rangle} \left(-1 - 2\psi^{\langle F \rangle} \left(-\delta^{\langle F \rangle} + I_{ss}^{\langle F \rangle} K_{ss}^{\langle F \rangle -1} \right) \right) \left(-\delta^{\langle F \rangle} + I_{ss}^{\langle F \rangle} K_{ss}^{\langle F \rangle -1} \right) \quad (8.12)$$

$$\beta \left(\lambda_{ss}^{c\langle H \rangle} \left(r_{ss}^{\langle H \rangle} - \psi^{\langle H \rangle} \left(-\delta^{\langle H \rangle} + I_{ss}^{\langle H \rangle} K_{ss}^{\langle H \rangle -1} \right)^2 + 2\psi^{\langle H \rangle} I_{ss}^{\langle H \rangle} K_{ss}^{\langle H \rangle -1} \left(-\delta^{\langle H \rangle} + I_{ss}^{\langle H \rangle} K_{ss}^{\langle H \rangle -1} \right) \right) - \lambda_{ss}^{c\langle H \rangle} \left(-1 - 2\psi^{\langle H \rangle} \left(-\delta^{\langle H \rangle} + I_{ss}^{\langle H \rangle} K_{ss}^{\langle H \rangle -1} \right) \right) \left(1 - \delta^{\langle H \rangle} \right) \right) + \lambda_{ss}^{c\langle H \rangle} \left(-1 - 2\psi^{\langle H \rangle} \left(-\delta^{\langle H \rangle} + I_{ss}^{\langle H \rangle} K_{ss}^{\langle H \rangle -1} \right) \right) \left(-\delta^{\langle H \rangle} + I_{ss}^{\langle H \rangle} K_{ss}^{\langle H \rangle -1} \right) \quad (8.13)$$

$$\lambda_{ss}^{c\langle F \rangle} W_{ss}^{\langle F \rangle} + (-1 + \mu) C_{ss}^{\langle F \rangle \mu} \left(1 - H_{ss}^{\langle F \rangle}\right)^{-\mu} \left(C_{ss}^{\langle F \rangle \mu} \left(1 - H_{ss}^{\langle F \rangle}\right)^{1-\mu}\right)^{-\eta} = 0 \quad (8.14)$$

$$\lambda_{ss}^{c\langle H \rangle} W_{ss}^{\langle H \rangle} + (-1 + \mu) C_{ss}^{\langle H \rangle \mu} \left(1 - H_{ss}^{\langle H \rangle}\right)^{-\mu} \left(C_{ss}^{\langle H \rangle \mu} \left(1 - H_{ss}^{\langle H \rangle}\right)^{1-\mu}\right)^{-\eta} = 0 \quad (8.15)$$

$$-G_{ss}^{d\langle F \rangle} + \phi^{G\langle F \rangle} G_{ss}^{d\langle F \rangle} = 0 \quad (8.16)$$

$$-G_{ss}^{d\langle H \rangle} + \phi^{G\langle H \rangle} G_{ss}^{d\langle H \rangle} = 0 \quad (8.17)$$

$$I_{ss}^{\langle F \rangle} - K_{ss}^{\langle F \rangle} + K_{ss}^{\langle F \rangle} \left(1 - \delta^{\langle F \rangle}\right) = 0 \quad (8.18)$$

$$I_{ss}^{\langle H \rangle} - K_{ss}^{\langle H \rangle} + K_{ss}^{\langle H \rangle} \left(1 - \delta^{\langle H \rangle}\right) = 0 \quad (8.19)$$

$$U_{ss}^{\langle F \rangle} - \beta U_{ss}^{\langle F \rangle} - (1 - \eta)^{-1} \left(C_{ss}^{\langle F \rangle \mu} \left(1 - H_{ss}^{\langle F \rangle}\right)^{1-\mu}\right)^{1-\eta} = 0 \quad (8.20)$$

$$U_{ss}^{\langle H \rangle} - \beta U_{ss}^{\langle H \rangle} - (1 - \eta)^{-1} \left(C_{ss}^{\langle H \rangle \mu} \left(1 - H_{ss}^{\langle H \rangle}\right)^{1-\mu}\right)^{1-\eta} = 0 \quad (8.21)$$

$$-C_{ss}^{\langle F \rangle} - G_{ss}^{d\langle F \rangle} - I_{ss}^{\langle F \rangle} - TR_{ss}^{\langle H \rangle} + Y_{ss}^{\langle F \rangle} - \psi^{\langle F \rangle} K_{ss}^{\langle F \rangle} \left(-\delta^{\langle F \rangle} + I_{ss}^{\langle F \rangle} K_{ss}^{\langle F \rangle -1}\right)^2 = 0 \quad (8.22)$$

$$-C_{ss}^{\langle H \rangle} - G_{ss}^{d\langle H \rangle} - I_{ss}^{\langle H \rangle} + TR_{ss}^{\langle H \rangle} + Y_{ss}^{\langle H \rangle} - \psi^{\langle H \rangle} K_{ss}^{\langle H \rangle} \left(-\delta^{\langle H \rangle} + I_{ss}^{\langle H \rangle} K_{ss}^{\langle H \rangle -1}\right)^2 = 0 \quad (8.23)$$

9 Steady-state values

	Steady-state value
$\lambda^{c(F)}$	0.3934
$\lambda^{c(H)}$	0.3934
$r^{(F)}$	0.0351
$r^{(H)}$	0.0351
$C^{(F)}$	0.9578
$C^{(H)}$	0.9578
$G^{d(F)}$	0
$G^{d(H)}$	0
$H^{(F)}$	0.2645
$H^{(H)}$	0.2645
$I^{(F)}$	0.3816
$I^{(H)}$	0.3816
$K^{(F)}$	15.2627
$K^{(H)}$	15.2627
$TR^{(H)}$	0
$U^{(F)}$	-125.6048
$U^{(H)}$	-125.6048
$W^{(F)}$	3.0384
$W^{(H)}$	3.0384
$Y^{(F)}$	1.3393
$Y^{(H)}$	1.3393
$Z^{(F)}$	1
$Z^{(H)}$	1

10 The solution of the 1st order perturbation

Matrix P

$$\begin{matrix}
 G_t^{d(F)} \\
 G_t^{d(H)} \\
 K_t^{(F)} \\
 K_t^{(H)} \\
 Z_t^{(F)} \\
 Z_t^{(H)}
 \end{matrix}
 \begin{pmatrix}
 G_{t-1}^{d(F)} & G_{t-1}^{d(H)} & K_{t-1}^{(F)} & K_{t-1}^{(H)} & Z_{t-1}^{(F)} & Z_{t-1}^{(H)} \\
 0.95 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0.95 & 0 & 0 & 0 & 0 \\
 -0.1542 & -0.1542 & 0.9454 & 0.0244 & 2.2856 & -1.0704 \\
 -0.1542 & -0.1542 & 0.0244 & 0.9454 & -1.0704 & 2.2856 \\
 0 & 0 & 0 & 0 & 0.95 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0.95
 \end{pmatrix}$$

Matrix Q

$$\begin{matrix}
 G^{d(F)} \\
 G^{d(H)} \\
 K^{(F)} \\
 K^{(H)} \\
 Z^{(F)} \\
 Z^{(H)}
 \end{matrix}
 \begin{pmatrix}
 \epsilon^{Z^{(F)}} & \epsilon^{Z^{(H)}} & \epsilon^{G^{(F)}} & \epsilon^{G^{(H)}} \\
 0 & 0 & 1 & 0 \\
 0 & 0 & 0 & 1 \\
 2.4059 & -1.1267 & -0.1623 & -0.1623 \\
 -1.1267 & 2.4059 & -0.1623 & -0.1623 \\
 1 & 0 & 0 & 0 \\
 0 & 1 & 0 & 0
 \end{pmatrix}$$

Matrix R

$$\begin{array}{l}
 \lambda_t^{c(F)} \\
 \lambda_t^{c(H)} \\
 r_t^{(F)} \\
 r_t^{(H)} \\
 C_t^{(F)} \\
 C_t^{(H)} \\
 H_t^{(F)} \\
 H_t^{(H)} \\
 I_t^{(F)} \\
 I_t^{(H)} \\
 TR_t^{(H)} \\
 U_t^{(F)} \\
 U_t^{(H)} \\
 W_t^{(F)} \\
 W_t^{(H)} \\
 Y_t^{(F)} \\
 Y_t^{(H)}
 \end{array}
 \begin{pmatrix}
 G_{t-1}^{d(F)} & G_{t-1}^{d(H)} & K_{t-1}^{(F)} & K_{t-1}^{(H)} & Z_{t-1}^{(F)} & Z_{t-1}^{(H)} \\
 0.1022 & 0.1022 & -0.0091 & -0.0091 & -0.1072 & -0.1072 \\
 0.1022 & 0.1022 & -0.0091 & -0.0091 & -0.1072 & -0.1072 \\
 0.0044 & 0.0044 & -0.0012 & -0.0004 & 0.0497 & -0.0046 \\
 0.0044 & 0.0044 & -0.0004 & -0.0012 & -0.0046 & 0.0497 \\
 -0.1525 & -0.1525 & 0.0187 & 0.0136 & 0.3448 & 0.1599 \\
 -0.1525 & -0.1525 & 0.0136 & 0.0187 & 0.1599 & 0.3448 \\
 0.0554 & 0.0554 & 0.0023 & -0.0049 & 0.2054 & -0.0581 \\
 0.0554 & 0.0554 & -0.0049 & 0.0023 & -0.0581 & 0.2054 \\
 -0.1542 & -0.1542 & -0.0296 & 0.0244 & 2.2856 & -1.0704 \\
 -0.1542 & -0.1542 & 0.0244 & -0.0296 & -1.0704 & 2.2856 \\
 -0.475 & 0.475 & 0.053 & -0.053 & -0.7338 & 0.7338 \\
 -3.1408 & -3.1408 & 0.1608 & 0.2366 & 0.053 & 8.3603 \\
 -3.1408 & -3.1408 & 0.2366 & 0.1608 & 8.3603 & 0.053 \\
 -0.2547 & -0.2547 & 0.0689 & 0.0227 & 1.9424 & 0.2672 \\
 -0.2547 & -0.2547 & 0.0227 & 0.0689 & 0.2672 & 1.9424 \\
 0.1684 & 0.1684 & 0.0422 & -0.015 & 1.8966 & -0.1767 \\
 0.1684 & 0.1684 & -0.015 & 0.0422 & -0.1767 & 1.8966
 \end{pmatrix}$$

Matrix S

$$\begin{array}{l}
 \lambda^{c(F)} \\
 \lambda^{c(H)} \\
 r^{(F)} \\
 r^{(H)} \\
 C^{(F)} \\
 C^{(H)} \\
 H^{(F)} \\
 H^{(H)} \\
 I^{(F)} \\
 I^{(H)} \\
 TR^{(H)} \\
 U^{(F)} \\
 U^{(H)} \\
 W^{(F)} \\
 W^{(H)} \\
 Y^{(F)} \\
 Y^{(H)}
 \end{array}
 \begin{pmatrix}
 \epsilon^{Z(F)} & \epsilon^{Z(H)} & \epsilon^{G(F)} & \epsilon^{G(H)} \\
 -0.1128 & -0.1128 & 0.1075 & 0.1075 \\
 -0.1128 & -0.1128 & 0.1075 & 0.1075 \\
 0.0523 & -0.0049 & 0.0046 & 0.0046 \\
 -0.0049 & 0.0523 & 0.0046 & 0.0046 \\
 0.3629 & 0.1683 & -0.1605 & -0.1605 \\
 0.1683 & 0.3629 & -0.1605 & -0.1605 \\
 0.2163 & -0.0612 & 0.0583 & 0.0583 \\
 -0.0612 & 0.2163 & 0.0583 & 0.0583 \\
 2.4059 & -1.1267 & -0.1623 & -0.1623 \\
 -1.1267 & 2.4059 & -0.1623 & -0.1623 \\
 -0.7724 & 0.7724 & -0.5 & 0.5 \\
 0.0557 & 8.8003 & -3.3061 & -3.3061 \\
 8.8003 & 0.0557 & -3.3061 & -3.3061 \\
 2.0446 & 0.2812 & -0.2681 & -0.2681 \\
 0.2812 & 2.0446 & -0.2681 & -0.2681 \\
 1.9964 & -0.186 & 0.1773 & 0.1773 \\
 -0.186 & 1.9964 & 0.1773 & 0.1773
 \end{pmatrix}$$

11 Model statistics

11.1 Basic statistics

	Steady-state value	Std. dev.	Variance	Loglin
$r^{(H)}$	0.0351	0.0051	0	N
$C^{(H)}$	0.9578	0.034	0.0012	N
$G^{d(H)}$	0	0.0922	0.0085	N
$H^{(H)}$	0.2645	0.0249	0.0006	N
$I^{(H)}$	0.3816	0.2411	0.0581	N
$K^{(H)}$	15.2627	0.8242	0.6794	N
$TR^{(H)}$	0	0.1586	0.0252	N
$U^{(H)}$	-125.6048	0.7839	0.6144	N
$W^{(H)}$	3.0384	0.1864	0.0348	N
$Y^{(H)}$	1.3393	0.2022	0.0409	N
$Z^{(H)}$	1	0.0922	0.0085	N

11.2 Correlation matrix

	$r^{(H)}$	$C^{(H)}$	$G^{d(H)}$	$H^{(H)}$	$I^{(H)}$	$K^{(H)}$	$TR^{(H)}$	$U^{(H)}$	$W^{(H)}$	$Y^{(H)}$	$Z^{(H)}$
$r^{(H)}$	1	0.659	0.554	0.918	0.888	0.193	0.623	-0.274	0.889	0.934	0.983
$C^{(H)}$		1	0.06	0.562	0.592	0.393	0.241	0.314	0.889	0.712	0.769
$G^{d(H)}$			1	0.625	0.391	0.151	0.493	-0.39	0.38	0.545	0.5
$H^{(H)}$				1	0.813	0.512	0.469	-0.445	0.878	0.981	0.927
$I^{(H)}$					1	0.231	0.817	-0.501	0.792	0.829	0.874
$K^{(H)}$						1	-0.15	-0.259	0.511	0.527	0.325
$TR^{(H)}$							1	-0.534	0.398	0.454	0.55
$U^{(H)}$								1	-0.064	-0.303	-0.197
$W^{(H)}$									1	0.954	0.958
$Y^{(H)}$										1	0.968
$Z^{(H)}$											1

11.3 Cross correlations with the reference variable ($Y^{(H)}$)

	$\sigma[\cdot]$ rel. to $\sigma[Y^{(H)}]$	$Y_{t-4}^{(H)}$	$Y_{t-3}^{(H)}$	$Y_{t-2}^{(H)}$	$Y_{t-1}^{(H)}$	$Y_t^{(H)}$	$Y_{t+1}^{(H)}$	$Y_{t+2}^{(H)}$	$Y_{t+3}^{(H)}$	$Y_{t+4}^{(H)}$
$r_t^{(H)}$	0.025	0.28	0.415	0.57	0.744	0.934	0.583	0.301	0.081	-0.085
$C_t^{(H)}$	0.168	0.053	0.174	0.324	0.503	0.712	0.562	0.425	0.301	0.192
$G_t^{d(H)}$	0.456	0.118	0.202	0.301	0.415	0.545	0.373	0.229	0.113	0.021
$H_t^{(H)}$	0.123	0.187	0.344	0.529	0.741	0.981	0.725	0.501	0.309	0.148
$I_t^{(H)}$	1.192	0.283	0.401	0.532	0.676	0.829	0.5	0.239	0.037	-0.113
$K_t^{(H)}$	4.076	-0.182	-0.06	0.097	0.292	0.527	0.661	0.714	0.707	0.656
$TR_t^{(H)}$	0.785	0.293	0.339	0.384	0.423	0.454	0.145	-0.081	-0.237	-0.336
$U_t^{(H)}$	3.876	-0.155	-0.196	-0.236	-0.273	-0.303	-0.189	-0.095	-0.021	0.037
$W_t^{(H)}$	0.922	0.134	0.291	0.479	0.701	0.954	0.726	0.522	0.345	0.193
$Y_t^{(H)}$	1	0.171	0.333	0.525	0.748	1	0.748	0.525	0.333	0.171
$Z_t^{(H)}$	0.456	0.229	0.377	0.549	0.747	0.968	0.656	0.398	0.188	0.024

11.4 Autocorrelations

	Lag 1	Lag 2	Lag 3	Lag 4
$r^{(H)}$	0.704	0.456	0.254	0.093
$C^{(H)}$	0.75	0.529	0.338	0.177
$G^d{}^{(H)}$	0.713	0.471	0.271	0.11
$H^{(H)}$	0.748	0.525	0.333	0.171
$I^{(H)}$	0.698	0.447	0.244	0.083
$K^{(H)}$	0.956	0.852	0.709	0.546
$TR^{(H)}$	0.72	0.482	0.283	0.122
$U^{(H)}$	0.731	0.499	0.303	0.142
$W^{(H)}$	0.748	0.526	0.334	0.173
$Y^{(H)}$	0.748	0.525	0.333	0.171
$Z^{(H)}$	0.713	0.471	0.271	0.11

11.5 Variance decomposition

	$\epsilon^Z{}^{(F)}$	$\epsilon^Z{}^{(H)}$	$\epsilon^G{}^{(F)}$	$\epsilon^G{}^{(H)}$
$r^{(H)}$	0.002	0.987	0.005	0.005
$C^{(H)}$	0.059	0.657	0.142	0.142
$G^d{}^{(H)}$	0	0.25	0	0.75
$H^{(H)}$	0.051	0.88	0.035	0.035
$I^{(H)}$	0.214	0.78	0.003	0.003
$K^{(H)}$	0.208	0.785	0.003	0.003
$TR^{(H)}$	0.437	0.437	0.063	0.063
$U^{(H)}$	0.733	0.041	0.113	0.113
$W^{(H)}$	0.005	0.968	0.014	0.014
$Y^{(H)}$	0.021	0.969	0.005	0.005
$Z^{(H)}$	0	1	0	0

12 Impulse response functions

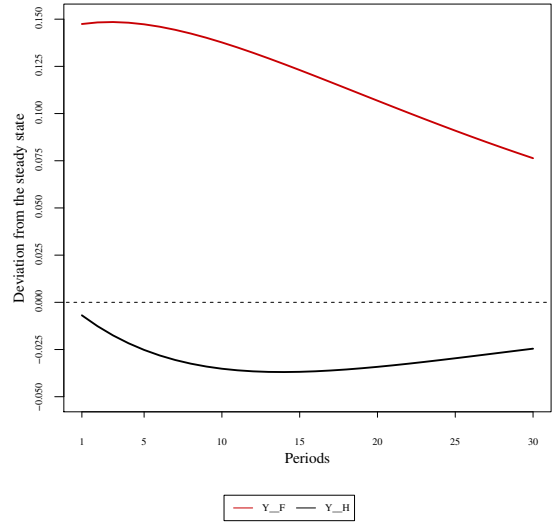
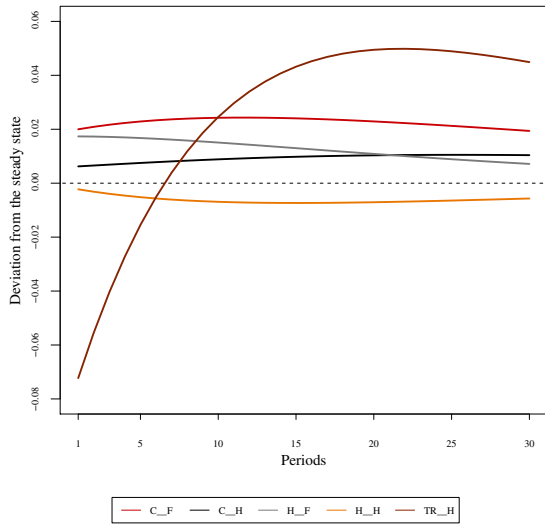


Figure 1: Impulse responses $(C^{(F)}, C^{(H)}, H^{(F)}, H^{(H)}, TR^{(H)})$ to $\epsilon^{Z^{(F)}}$ shock

Figure 2: Impulse responses $(Y^{(F)}, Y^{(H)})$ to $\epsilon^{Z^{(F)}}$ shock

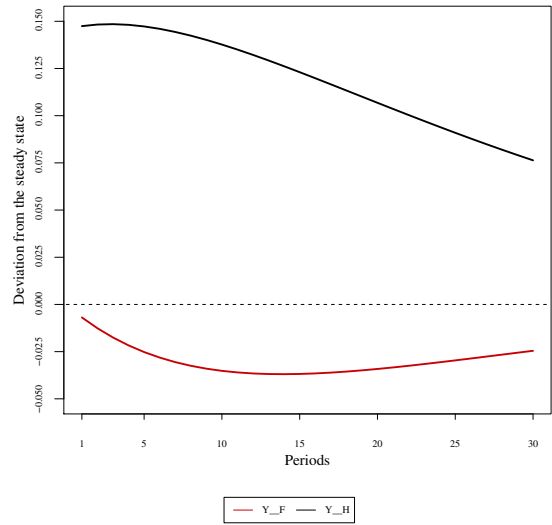
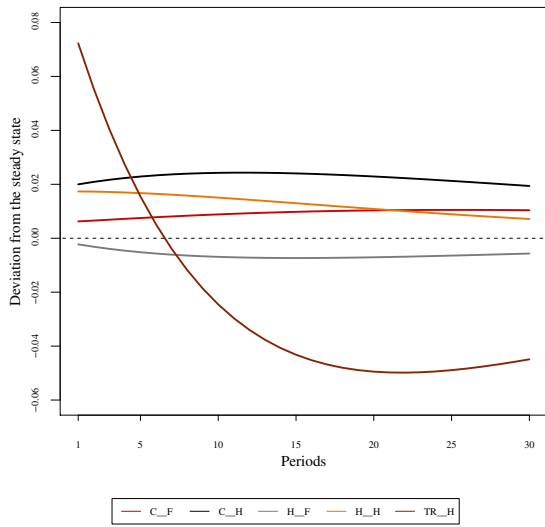


Figure 3: Impulse responses $(C^{(F)}, C^{(H)}, H^{(F)}, H^{(H)}, TR^{(H)})$ to $\epsilon^{Z^{(H)}}$ shock

Figure 4: Impulse responses $(Y^{(F)}, Y^{(H)})$ to $\epsilon^{Z^{(H)}}$ shock