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1 CONSUMER

1.1 Optimisation problem

$$\max_{K_t^{\mathrm{s}}, C_t, L_t^{\mathrm{s}}, I_t} U_t = \beta \mathrm{E}_t \left[U_{t+1}\right] + (1-\eta)^{-1} \left({C_t}^{\mu}(1-L_t^{\mathrm{s}})^{1-\mu}\right)^{1-\eta} \tag{1.1}$$

s.t. :

$$C_t + I_t = \pi_t + K_{t-1}^{\mathrm{s}} r_t + L_t^{\mathrm{s}} W_t - \psi K_{t-1}^{\mathrm{s}} \left(-\delta + {K_{t-1}^{\mathrm{s}}}^{-1} I_t\right)^2 \quad (\lambda_t^{\mathrm{c}}) \tag{1.2}$$

$$K_t^{\mathrm{s}} = I_t + K_{t-1}^{\mathrm{s}} (1-\delta) \quad \left(\lambda_t^{\mathrm{CONSUMER}^2}\right) \tag{1.3}$$

1.2 First order conditions

$$-\lambda_t^{\mathrm{CONSUMER}^2} + \beta \left((1-\delta)\, \mathrm{E}_t \left[\lambda_{t+1}^{\mathrm{CONSUMER}^2}\right] + \mathrm{E}_t \left[\lambda_{t+1}^{\mathrm{c}} \left(r_{t+1} - \psi \left(-\delta + K_t^{\mathrm{s}-1} I_{t+1}\right)^2 + 2\psi K_t^{\mathrm{s}-1} I_{t+1} \left(-\delta + K_t^{\mathrm{s}-1} I_{t+1}\right)\right)\right]\right) = 0 \quad (K_t^{\mathrm{s}}) \tag{1.4}$$

$$-\lambda_t^{\mathrm{c}} + \mu {C_t}^{-1+\mu} (1-L_t^{\mathrm{s}})^{1-\mu} \left({C_t}^{\mu}(1-L_t^{\mathrm{s}})^{1-\mu}\right)^{-\eta} = 0 \quad (C_t) \tag{1.5}$$

$$\lambda_t^{\mathrm{c}} W_t + (-1+\mu)\, {C_t}^{\mu} (1-L_t^{\mathrm{s}})^{-\mu} \left({C_t}^{\mu}(1-L_t^{\mathrm{s}})^{1-\mu}\right)^{-\eta} = 0 \quad (L_t^{\mathrm{s}}) \tag{1.6}$$

$$\lambda_t^{\mathrm{CONSUMER}^2} + \lambda_t^{\mathrm{c}} \left(-1 - 2\psi \left(-\delta + {K_{t-1}^{\mathrm{s}}}^{-1} I_t\right)\right) = 0 \quad (I_t) \tag{1.7}$$

2 FIRM

2.1 Optimisation problem

$$\max_{K_t^{\mathrm{d}}, L_t^{\mathrm{d}}, Y_t, \pi_t} \Pi_t = \pi_t \tag{2.1}$$

s.t. :

$$Y_t = Z_t {K_t^{\mathrm{d}}}^{\alpha} {L_t^{\mathrm{d}}}^{1-\alpha} \quad \left(\lambda_t^{\mathrm{FIRM}^1}\right) \tag{2.2}$$

$$\pi_t = Y_t - L_t^{\mathrm{d}} W_t - r_t K_t^{\mathrm{d}} \quad \left(\lambda_t^{\mathrm{FIRM}^2}\right) \tag{2.3}$$

2.2 First order conditions

$$-\lambda_t^{\mathrm{FIRM}^2} r_t + \alpha \lambda_t^{\mathrm{FIRM}^1} Z_t {K_t^{\mathrm{d}}}^{-1+\alpha} {L_t^{\mathrm{d}}}^{1-\alpha} = 0 \quad \left(K_t^{\mathrm{d}}\right) \tag{2.4}$$

$$-\lambda_t^{\mathrm{FIRM}^2} W_t + \lambda_t^{\mathrm{FIRM}^1} Z_t \left(1-\alpha\right) {K_t^{\mathrm{d}}}^{\alpha} {L_t^{\mathrm{d}}}^{-\alpha} = 0 \quad \left(L_t^{\mathrm{d}}\right) \tag{2.5}$$

$$-\lambda_t^{\mathrm{FIRM}^1} + \lambda_t^{\mathrm{FIRM}^2} = 0 \quad \left(Y_t\right) \tag{2.6}$$

$$1 - \lambda_t^{\mathrm{FIRM}^2} = 0 \quad \left(\pi_t\right) \tag{2.7}$$

2.3 First order conditions after reduction

$$-r_t + \alpha Z_t {K_t^{\mathrm{d}}}^{-1+\alpha} {L_t^{\mathrm{d}}}^{1-\alpha} = 0 \quad \left(K_t^{\mathrm{d}}\right) \tag{2.8}$$

$$-W_t + Z_t \left(1-\alpha\right) {K_t^{\mathrm{d}}}^{\alpha} {L_t^{\mathrm{d}}}^{-\alpha} = 0 \quad \left(L_t^{\mathrm{d}}\right) \tag{2.9}$$

3 EQUILIBRIUM

3.1 Identities

$$K_t^{\mathrm{d}} = K_{t-1}^{\mathrm{s}} \tag{3.1}$$

$$L_t^{\mathrm{d}} = L_t^{\mathrm{s}} \tag{3.2}$$

4 EXOG

4.1 Identities

$$Z_t = e^{\epsilon_t^{\mathrm{Z}} + \phi \log Z_{t-1}} \tag{4.1}$$

5 Equilibrium relationships (after reduction)

$$-r_t + \alpha Z_t {K_{t-1}^{\mathrm{s}}}^{-1+\alpha} {L_t^{\mathrm{s}}}^{1-\alpha} = 0 \tag{5.1}$$

$$-W_t + Z_t \left(1-\alpha\right) {K_{t-1}^{\mathrm{s}}}^{\alpha} {L_t^{\mathrm{s}}}^{-\alpha} = 0 \tag{5.2}$$

$$-Y_t + Z_t {K_{t-1}^{\mathrm{s}}}^{\alpha} {L_t^{\mathrm{s}}}^{1-\alpha} = 0 \tag{5.3}$$

$$-Z_t + e^{\epsilon_t^{\mathrm{Z}} + \phi \log Z_{t-1}} = 0 \tag{5.4}$$

$$\beta \left(\mu \mathrm{E}_t \left[\left(r_{t+1} - \psi \left(-\delta + K_t^{\mathrm{s}-1} I_{t+1}\right)^2 + 2\psi K_t^{\mathrm{s}-1} I_{t+1} \left(-\delta + K_t^{\mathrm{s}-1} I_{t+1}\right)\right) {C_{t+1}}^{-1+\mu} \left(1 - L_{t+1}^{\mathrm{s}}\right)^{1-\mu} \left({C_{t+1}}^{\mu} \left(1 - L_{t+1}^{\mathrm{s}}\right)^{1-\mu}\right)^{-\eta}\right] - \mu \left(1-\delta\right) \mathrm{E}_t \left[\left(-1 - 2\psi \left(-\delta + K_t^{\mathrm{s}-1} I_{t+1}\right)\right) {C_{t+1}}^{-1}\right. \tag{5.5}$$

$$(-1+\mu)\, {C_t}^{\mu}(1-L_t^{\mathrm{s}})^{-\mu}\left({C_t}^{\mu}(1-L_t^{\mathrm{s}})^{1-\mu}\right)^{-\eta}+\mu W_t {C_t}^{-1+\mu}(1-L_t^{\mathrm{s}})^{1-\mu}\left({C_t}^{\mu}(1-L_t^{\mathrm{s}})^{1-\mu}\right)^{-\eta}=0 \tag{5.6}$$

$$I_t-K_t^{\mathrm{s}}+K_{t-1}^{\mathrm{s}}\,(1-\delta)=0 \tag{5.7}$$

$$U_t-\beta \mathrm{E}_t\left[U_{t+1}\right]-(1-\eta)^{-1}\left({C_t}^{\mu}(1-L_t^{\mathrm{s}})^{1-\mu}\right)^{1-\eta}=0 \tag{5.8}$$

$$-C_t-I_t+Y_t-\psi K_{t-1}^{\mathrm{s}}\left(-\delta+{K_{t-1}^{\mathrm{s}}}^{-1}I_t\right)^2=0 \tag{5.9}$$

6 Steady state relationships (after reduction)

$$-r_{\mathrm{ss}}+\alpha Z_{\mathrm{ss}}{K_{\mathrm{ss}}^{\mathrm{s}}}^{-1+\alpha}{L_{\mathrm{ss}}^{\mathrm{s}}}^{1-\alpha}=0 \tag{6.1}$$

$$-W_{\mathrm{ss}}+Z_{\mathrm{ss}}\,(1-\alpha)\,{K_{\mathrm{ss}}^{\mathrm{s}}}^{\alpha}{L_{\mathrm{ss}}^{\mathrm{s}}}^{-\alpha}=0 \tag{6.2}$$

$$-Y_{\mathrm{ss}}+Z_{\mathrm{ss}}{K_{\mathrm{ss}}^{\mathrm{s}}}^{\alpha}{L_{\mathrm{ss}}^{\mathrm{s}}}^{1-\alpha}=0 \tag{6.3}$$

$$-Z_{\mathrm{ss}}+e^{\phi \log Z_{\mathrm{ss}}}=0 \tag{6.4}$$

$$\beta\left(\mu\left(r_{\mathrm{ss}}-\psi\left(-\delta+I_{\mathrm{ss}}{K_{\mathrm{ss}}^{\mathrm{s}}}^{-1}\right)^2+2\psi I_{\mathrm{ss}}{K_{\mathrm{ss}}^{\mathrm{s}}}^{-1}\left(-\delta+I_{\mathrm{ss}}{K_{\mathrm{ss}}^{\mathrm{s}}}^{-1}\right)\right){C_{\mathrm{ss}}}^{-1+\mu}(1-L_{\mathrm{ss}}^{\mathrm{s}})^{1-\mu}\left({C_{\mathrm{ss}}}^{\mu}(1-L_{\mathrm{ss}}^{\mathrm{s}})^{1-\mu}\right)^{-\eta}-\mu\left(-1-2\psi\left(-\delta+I_{\mathrm{ss}}{K_{\mathrm{ss}}^{\mathrm{s}}}^{-1}\right)\right)(1-\delta)\,{C_{\mathrm{ss}}}^{-1+\mu}(1-L_{\mathrm{ss}}^{\mathrm{s}})^{1-\mu}\left({C_{\mathrm{ss}}}^{\mu}(1\right. \tag{6.5}$$

$$(-1+\mu)\,{C_{\mathrm{ss}}}^{\mu}(1-L_{\mathrm{ss}}^{\mathrm{s}})^{-\mu}\left({C_{\mathrm{ss}}}^{\mu}(1-L_{\mathrm{ss}}^{\mathrm{s}})^{1-\mu}\right)^{-\eta}+\mu W_{\mathrm{ss}}{C_{\mathrm{ss}}}^{-1+\mu}(1-L_{\mathrm{ss}}^{\mathrm{s}})^{1-\mu}\left({C_{\mathrm{ss}}}^{\mu}(1-L_{\mathrm{ss}}^{\mathrm{s}})^{1-\mu}\right)^{-\eta}=0 \tag{6.6}$$

$$I_{\mathrm{ss}}-K_{\mathrm{ss}}^{\mathrm{s}}+K_{\mathrm{ss}}^{\mathrm{s}}\,(1-\delta)=0 \tag{6.7}$$

$$U_{\mathrm{ss}}-\beta U_{\mathrm{ss}}-(1-\eta)^{-1}\left({C_{\mathrm{ss}}}^{\mu}(1-L_{\mathrm{ss}}^{\mathrm{s}})^{1-\mu}\right)^{1-\eta}=0 \tag{6.8}$$

$$-C_{\mathrm{ss}}-I_{\mathrm{ss}}+Y_{\mathrm{ss}}-\psi K_{\mathrm{ss}}^{\mathrm{s}}\left(-\delta+I_{\mathrm{ss}}{K_{\mathrm{ss}}^{\mathrm{s}}}^{-1}\right)^2=0 \tag{6.9}$$

7 Calibrating equations

$$-0.36Y_{\mathrm{ss}}+r_{\mathrm{ss}}K_{\mathrm{ss}}^{\mathrm{s}}=0 \tag{7.1}$$

8 Parameter settings

$$\beta=0.99 \tag{8.1}$$

$$\delta=0.025 \tag{8.2}$$

$$\eta=2 \tag{8.3}$$

$$\mu=0.3 \tag{8.4}$$

$$\phi=0.95 \tag{8.5}$$

$$\psi=0.8 \tag{8.6}$$

9 Steady-state values

	Steady-state value
r	0.0351
C	0.7422
I	0.2559
K^{s}	10.2368
L^{s}	0.2695
U	-136.2372
W	2.3706
Y	0.9981
Z	1

10 The solution of the 1st order perturbation

Matrix P

$$\begin{array}{c} \\ K_t^{\mathrm{s}} \\ Z_t \end{array} \begin{array}{c} \begin{array}{cc} K_{t-1}^{\mathrm{s}} & Z_{t-1} \end{array} \\ \begin{pmatrix} 0.9658 & 0.0863 \\ 0 & 0.95 \end{pmatrix} \end{array}$$

Matrix Q

$$\begin{array}{c} \\ K^{\mathrm{s}} \\ Z \end{array} \begin{array}{c} \epsilon^{\mathrm{Z}} \\ \begin{pmatrix} 0.0908 \\ 1 \end{pmatrix} \end{array}$$

Matrix R

$$\begin{array}{c} \\ r_t \\ C_t \\ I_t \\ L_t^{\mathrm{s}} \\ U_t \\ W_t \\ Y_t \end{array} \begin{array}{c} \begin{array}{cc} K_{t-1}^{\mathrm{s}} & Z_{t-1} \end{array} \\ \begin{pmatrix} -0.7408 & 1.2972 \\ 0.4748 & 0.5545 \\ -0.3661 & 3.4511 \\ -0.1575 & 0.5426 \\ 0.0418 & 0.0644 \\ 0.4167 & 0.7547 \\ 0.2592 & 1.2972 \end{pmatrix} \end{array}$$

Matrix S

$$\begin{array}{c} \\ r \\ C \\ I \\ L^{\mathrm{s}} \\ U \\ W \\ Y \end{array} \begin{array}{c} \epsilon^{\mathrm{Z}} \\ \begin{pmatrix} 1.3655 \\ 0.5837 \\ 3.6328 \\ 0.5711 \\ 0.0678 \\ 0.7944 \\ 1.3655 \end{pmatrix} \end{array}$$

11 Model statistics

11.1 Basic statistics

	Steady-state value	Std. dev.	Variance	Loglin
r	0.0351	0.1814	0.0329	Y
C	0.7422	0.0783	0.0061	Y
I	0.2559	0.4741	0.2248	Y
K^{s}	10.2368	0.0422	0.0018	Y
L^{s}	0.2695	0.0749	0.0056	Y
U	-136.2372	0.009	0.0001	Y
W	2.3706	0.1047	0.011	Y
Y	0.9981	0.1781	0.0317	Y
Z	1	0.1303	0.017	Y

11.2 Correlation matrix

	r	C	I	K^{s}	L^{s}	U	W	Y	Z
r	1	0.908	0.99	0.09	0.996	0.932	0.942	0.973	0.985
C		1	0.958	0.498	0.94	0.998	0.996	0.981	0.967
I			1	0.228	0.998	0.974	0.98	0.996	0.999
K^{s}				1	0.173	0.445	0.418	0.319	0.26
L^{s}					1	0.959	0.967	0.989	0.996
U						1	1	0.991	0.981
W							1	0.994	0.986
Y								1	0.998
Z									1

11.3 Cross correlations with the reference variable (Y)

	$\sigma[\cdot]$ rel. to $\sigma[Y]$	Y_{t-5}	Y_{t-4}	Y_{t-3}	Y_{t-2}	Y_{t-1}	Y_t	Y_{t+1}	Y_{t+2}	Y_{t+3}	Y_{t+4}	Y_{t+5}
r_t	1.018	0.109	0.228	0.373	0.545	0.745	0.973	0.631	0.353	0.132	-0.037	-0.161
C_t	0.439	-0.107	0.021	0.189	0.402	0.665	0.981	0.761	0.564	0.392	0.245	0.121
I_t	2.662	0.039	0.164	0.319	0.508	0.733	0.996	0.688	0.431	0.222	0.057	-0.07
K^{s}_t	0.237	-0.48	-0.422	-0.321	-0.17	0.04	0.319	0.504	0.612	0.66	0.659	0.623
L^{s}_t	0.42	0.067	0.19	0.341	0.524	0.74	0.989	0.666	0.401	0.187	0.019	-0.107
U_t	0.05	-0.077	0.052	0.218	0.428	0.684	0.991	0.751	0.54	0.359	0.207	0.081
W_t	0.588	-0.062	0.066	0.232	0.439	0.692	0.994	0.745	0.528	0.343	0.188	0.062
Y_t	1	-0.008	0.119	0.28	0.479	0.718	1	0.718	0.479	0.28	0.119	-0.008
Z_t	0.732	0.023	0.148	0.306	0.499	0.729	0.998	0.699	0.448	0.242	0.078	-0.049

11.4 Autocorrelations

	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5
r	0.71	0.466	0.266	0.104	-0.022
C	0.745	0.521	0.329	0.169	0.038
I	0.712	0.468	0.268	0.107	-0.019
K^{s}	0.96	0.863	0.728	0.572	0.408
L^{s}	0.71	0.466	0.265	0.103	-0.022
U	0.735	0.505	0.311	0.15	0.02
W	0.73	0.498	0.303	0.142	0.013
Y	0.718	0.479	0.28	0.119	-0.008
Z	0.713	0.471	0.271	0.11	-0.016

12 Impulse response functions

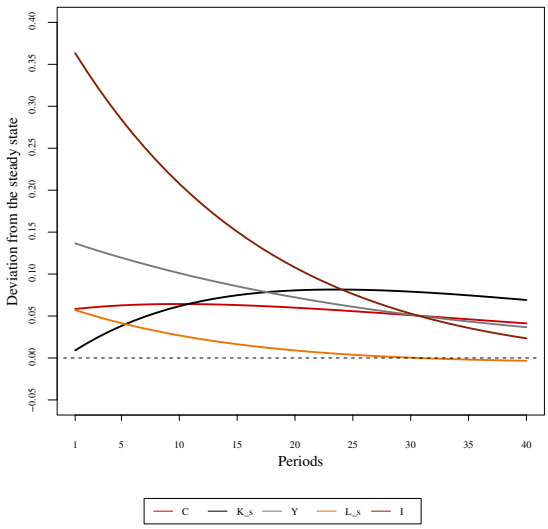

Figure 1: Impulse responses $(C, K^{\mathrm{s}}, Y, L^{\mathrm{s}}, I)$ to ϵ^{Z} shock

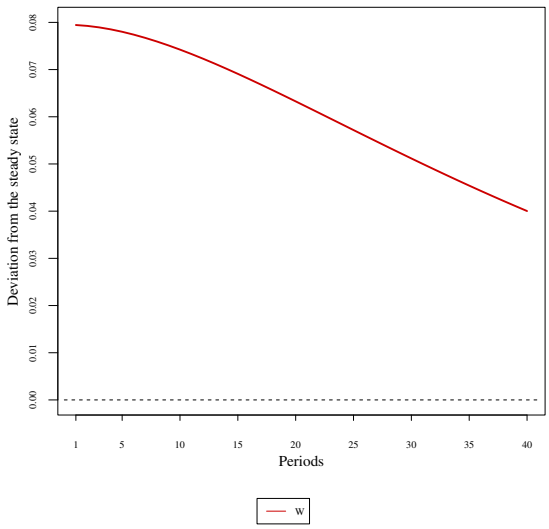

Figure 2: Impulse response (W) to ϵ^{Z} shock